Cycle Stealing:
A Probabilistic Model

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Background

The scenario is two machines between which jobs or tasks are shared. Machine A sends its jobs to machine B according to a *schedule* of time intervals. The issue is complicated by the fact that the work on machine B is lost upon the return of user B. The return of user B can be modeled with a random variable. Therefore, the problem is to generate a schedule which maximizes the amount of work done based on a given model for user B's return.

1 Proposal

My proposal is to work with this problem in the special cases that a user's return is modeled by *exponential* and *uniform* distributions. I will examine both distributions in discrete and continuous cases.

In the continuous case, the problem generalizes to a continuous recurrence relation for W_{opt}, the expected amount of work completed given an optimal schedule. This approach suggests the use of dynamic programming to solve the recurrence for the optimal schedule which maximizes W_{opt}. I will implement a dynamic program to solve for W_{opt} in Mathematica. In doing this, I hope to find out if this approach may be used for other distributions.

2 More Details

This section will elaborate on the model discussed above. First, let X be the random variable representing the interrupt, that is, the return of user B to his or her workstation. Let c be a constant representing overhead (this overhead could include transmission times for jobs as well as set up time for the jobs on machine B). Let $S = t_0, t_1, \ldots$ be a schedule of time intervals representing jobs. Then we can define $W(S, t)$ to be the work accomplished if user B returns at time t (assuming the schedule starts at time 0). $W(S,t)$ obeys

$$\sum_{i=0}^{\infty} t_i \ominus c$$

forall $t_i < t$. The $\ominus$ operator represents positive subtraction, also known as the *monus* operator. Furthermore, let W_{opt} be the worked accomplished by an optimal schedule S_{opt}. Then W_{opt} obeys the continuos recurrence relation

$$\begin{aligned} W_{opt}(0) &= \mathrm{E}[W(S_{opt}, X)] \\ &= \max_{c \leq t_0} \{(t_o \ominus c)(1 - F_X(t_0)) + (1 - F_X(t_0))W_{opt}(t_0)\} \end{aligned}$$

where F_X is the distribution function of random variable X. This is the recurrence which I will solve to find an optimal schedule for cycle stealing.

3 Information Sources

The original problem of stealing cycles was first presented to me by Dr. Arnold Rosenberg of the University of Massachusetts. His paper *On Optimal Strategies for Stealing Cycles* formalizes the problem so that it may be analyzed mathematically.

Most of the notation and models expressed above were shown to me by Dr. Lenny Heath in a weekly meeting of the Virginia Tech Theory Group.

My reference for probability theory comes from *Probability, Statistics, and Queueing Theory with Computer Science Applications, 2nd edition* by Arnold O. Allen.

Recurrence relations and their solutions are discussed in *Applied Combinatorics* by Fred S. Roberts.

Finally, an introduction to Mathematica programming is presented in *Mathematica: A System for Doing Mathematics by Computer, 2nd edition* by Stephen Wolfram.