Project Report: Cycle Stealing

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Abstract

This report documents and highlights the results of research of the problem known as cycle stealing [BCLR95]. Highlights include characterizations of optimal schedules modeled by both exponential and geometric distributions.

Introduction

Recall that the scenario is two machines between which jobs or tasks are shared. Machine A sends its jobs to machine B according to a schedule of time intervals. The issue is complicated by the fact that the work on machine B is lost upon the return of user B. The return of user B can be modeled with a random variable. Therefore, the problem is to generate a schedule which maximizes the amount of work done based on a given model for user B's return.

Using both the exponential and geometric distributions to model the interrupt time (i.e. the return of user B), I was able to determine what optimal schedules look like. Results for the exponential distribution are discussed in section 2 and those for the geometric distribution are discussed in section 3.

1 Definitions

First, let X be the random variable representing the interrupt, that is, the return of user B to his or her workstation. Let c be a constant representing overhead (this overhead could include transmission times for jobs as well as set up time for the jobs on machine B). Let $S = t_0, t_1, \ldots$ be a schedule of time intervals representing jobs. Then we can define $W(S, t)$ to be the work accomplished if user B returns at time t (assuming the schedule starts at time 0). $W(S, t)$ obeys

$$\sum_{i=0}^{\infty} t_i \ominus c$$

for all $t_i < t$. The $\ominus$ operator represents positive subtraction, also known as the *monus* operator. Furthermore, let W_{opt} be the worked accomplished by an optimal schedule S_{opt}. Then W_{opt} obeys the continuous recurrence relation

$$\begin{aligned} W_{opt}(0) &= \mathrm{E}[W(S_{opt}, X)] \\ &= \max_{c \leq t_0} \{(t_o \ominus c)(1 - F_X(t_0)) + (1 - F_X(t_0))W_{opt}(t_0)\} \end{aligned}$$

where F_X is the distribution function of random variable X. This is the recurrence which I will solve to find an optimal schedule for cycle stealing.

2 Exponential Distribution

When solving the recurrence with the exponential distribution, a closed form expression could not be found which would allow for simple evaluation of t due to the presence of the exponent function. However, both Taylor series approximations (of two and five terms) were used as well as numerical methods supplied by Mathematica's `NSolve` function. It turns out that the Taylor series approximation were very inaccurate, so the results found with `NSolve` were taken to be the real solutions.

Interpreting the results, we can see that t varies with α, which is the parameter to the exponential distribution, exponentially. The relationship is such that as α increases, and hence shortening the expected return time, t diminishes exponentially.

2.1 Optimal Work

The optimal t values found for a given α can substituted back into the solution to the recurrence to find the expected amount of work given an optimal solution. By doing this, we can see that the work varies with α the same way t did. That is, as α increases, the amount of work which can be done decreases exponentially.

2.2 Interpretation

The interpretation of these results is that as the expected return time of user B shortens, very little work can be accomplished. However, we can also see that as the return time lengthens to anything longer that the overhead c, then we begin to have our way with the system. That is to say we begin to accomplish a great deal of work. *Please see the Mathematica notebook that accompanies this report,* `exponential.ma`.

3 Geometric Distribution

The same method was followed here as with the exponential distribution. Again, after solving the recurrence, no closed form expression for t could be found due to the presence of the logarithm function. Numerical methods supplied by Mathematica's `FindRoot` function generated the desired results. *Please see the Mathematica notebook* `geometric.ma` *that accompanies this report.*

Examining the results, we see that as q increases, t increases. Remember that q is a parameter to the geometric distribution. For this problem, q is the probability that at any given time step, user B does *not* return. Therefore, q increasing means the likelihood of user B returning diminishes. Hence, the amount of work we can get done increases.

3.1 Optimal Work

Using the optimal t values, I solved for W_{opt} to find out what amount of work we could expect to achieve. The results are found at the end of the notebook `geometric.ma` in the section "Optimal Work".

It turns out that W_{opt} varies with q similarly to t. As q increases, the amount of work we can expect to get done also increases. This correspondence between W_{opt} and t was also seen in the exponential distribution.

4 Conclusions

The conclusions for each of the distributions may be summarized by the following statement. As the probability of user B's return decreases, we can expect to accomplish more work by cycle stealing. Although the overhead parameter c does not change this behavior, larger c values clearly make it harder to get work done by cycle stealing.

5 Future Work

Certainly there are a few things which can be done to extend this line of research. The first of which would be to examine other distributions. The geometric and exponential were chosen since their memoryless property simplifies the solution of the W_{opt} recurrence. Other distributions may be more appropriate.

References

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