Project Proposal

Graph Proofs Using the Larch Prover
CS 6104: Symbolic Computation

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Introduction

Graph theory is an important area of mathematics. Graphs represent many types of problems in many different application areas. With improving theorem proving systems, one might ask, "Can graph theoretic problems be proved using a theorem proving system?"

There are two aspects of automated theorem proving: representing the axiom system for the theory, and directing the theorem prover towards a proof of a theorem. The goals of this project are to represent the axioms of graph theory, explore what can be proved given the axioms, and prove theorems using an automated theorem prover.

Approach

Some work has already been done for representing graph theoretic properties in *Mathematica* [2]. Building on this work, and other representations to be researched in the library, representations of graph theory axioms for the *Larch Prover*(LP) will be created.

LP provides a number of proof techniques and utilities, such as Knuth-Bendix completion, for exploring the consequences of axiom systems. These tools will be used to gain information about what can and cannot be proved given an axiom system representation for LP.

Since there are many representations of graphs and axioms for graph theory, it is natural to explore the different representations and their advantages and disadvantages for automated theorem proving. One such representation of graphs is based on set theory. Because of this, an axiom system for set theory will be required. LP comes with such an axiom system and it will be used for this representation.

Goals

An obvious goal for this project is to create a LP based system for automated theorem proving of graph theorems. This system will contain many representations for graphs. Documentation for using the system will be included.

Another goal is to prove some simple theorems, such as the number of nodes in a full binary tree, the number of edges in a graph is equal to half the sum of the degrees of the nodes, and the Havel-Hakimi theorem about graphical degree sequences. This will show that the representations are sufficient to prove some basic properties about graphs.

If time permits, a more complex theorem will be explored and proved, such as the maximum flow-minimum cut theorem.

References

[1] G. Chartrand and O. R. Oellerman, *Applied and Algorithmic Graph Theory*, McGraw Hill, 1993.

[2] S. Skiena, *Implementing Discrete Mathematics*, Addison Wesley Publishing Company, 1990.