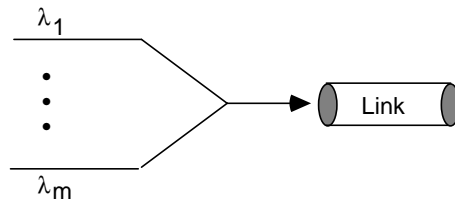


**Lecture 12**  
**CS/EE 5516 - Spring 1994**

Ch 4. Multiaccess Communication

In Ch 2,3 we compared TDM, SM:



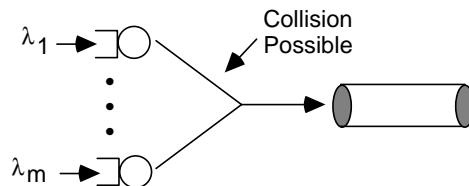
Q: When is TDM vs. SM preferable?

A: •  $\lambda_j$  small  $\Rightarrow$  SM (1 transmitter can use 100% of bandwidth)  
 want small delay  $\Rightarrow$  SM (wait  $m/2$  on avg. for TDM, wait 0 for SM)  
 want large  $\Rightarrow$  TDM (fair access)

Q: Can SM be implemented in a LAN or satellite net?

A: [LAN is wireless net, coax/fiber cable, U/S TP.]

NO: No central controller!  
 Best you can do is sync stations to use slots.



BG 4.2 Slotted Aloha

-Theory Underlying Ethernet

Model Assumptions

$m = \#$  nodes

1. Equal Length Packets (Discrete Time:  
 Sent on Slot Boundaries No xmit starts at 1.4)  
 Xmit is one slot in duration

Implications:

1. Permits discrete time model
2. Prohibits carrier sensing (ethernet), which takes  $<1$  slot time
2. Poisson Arrivals: Aggregate Rate  $\lambda$   
 Per node rate  $\lambda/m$
3. Collision occurs (whenever my daughter interrupts my wife)  
 or  
 Perfect Reception (no info on #colliding xmitters, etc)
  1. Ignores noise errors
  2. Prohibits receiver that can capture 1 xmit in face of collisions
  3. Prohibits cellular radio; all nodes "see" collision
4. 0,1,e Immediate Feedback  
 At end of slot every node learns if  
 0 pkts succeeded(idle)  
 1 pkt succeeded (idle)  
 e: error (collision)  
 $\Rightarrow$  Time delayed feedback (satellites) complicate analysis, but no fundamental problem
5. Retransmission of collisions in later timeslot until succeeds (node called backlogged)

6A. No buffering.

Backlogged node ignores arriving traffic  
=> delay w/GA is lowerbound

6B. • nodes:

There are • nodes, not m. Each arrival has its own node.  
=> Delay w/ 6B is upper bound.

#### BG 4.2.2 Slotted Aloha

University of Hawaii

**Q: What happens on collision? A:**

- xmit on next slot? No - Guarantees another collision
- wait random delay? Yes, but longer delays possible

Q: Use no buffering assumption (6a). For any  $\lambda$ :

[what will mean delay be?

[what will max throughput be? Near 0? Near 1? Near 1/2?

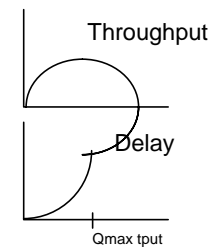
(Given  $\lambda$  and  $q_r$  = prob retrans in current slot)

Q: How would you find answer?

Little's law?

M/M/1 queue? M/M/m ?

Markov chain?



A: Safe answer (w/ poisson arrivals): Markov chain.

Q: What is 1st step to creating chain?

A: Choose system state (ideally, 1 integer)

Q: What should state be?

A: # backlogs (n):

(Class: Why not #customers in system?

Why not #backlogs + #new arrivals at slot start?)

What happens if no backlog, 1 arrival at slot? => "1"  
 (See table on next page)  
 Do we need 2D? (#backlog + #arrivals?)

We can use 1 dimension: # in system = # backlogged = n

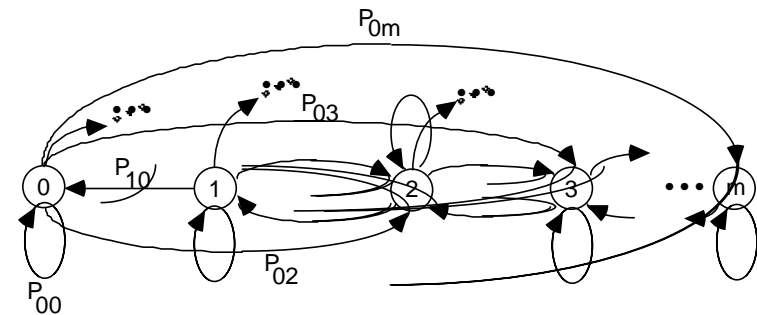
Consider transition from n to n + i. (one transition = one slot)

Q: What are legal values of i?

A:  $-1 \leq i \leq m-n$  for  $n > 0$   
 $-2 \leq i \leq m$  for  $n = 0$

List exhaustive cases based on i:

i	#arrivals	#backlogged that retransmit	result
-1	0	1	1
0	0	0	0
0	0	2,3,...,n	e
0	1	0	1
1	1	1,2,...,n	e
2,3,...,m-n	i	anything	e



$Q_a(i,n) = \text{Prob}[i \text{ unbacklogged nodes xmit in a slot}]$

$$= \binom{m-n}{i} (1-q_a)^{m-n-i} q_a^i$$

$Q_r(i,n) = \text{Prob}[i \text{ backlogged nodes xmit}]$

$$= \binom{n}{i} (1-q_r)^{n-i} q_r^i$$

Returning to table,  $P_{n,n+i} (-1 \leq i \leq m-n)$  can be calculated in terms of  $Q_a(i,n)$  and  $Q_r(i,n)$ .

Now we can derive transitive probabilities:

i	#arrivals	#backlogged that retransmit	result	Probability
-1	0	1	1	$P_{n,n-1} = Q_a(0,n)Q_r(1,n)$
0	0	0	0	$P_{n,n} = Q_a(0,n)[1-Q_r(1,n)] + Q_a(1,n)Q_r(0,n)$
0	0	2,3,...,n	e	
0	1	0	1	
1	1	1,2,...,n	e	$P_{n,n+1} = Q_a(1,n)[1-Q_r(0,n)]$
2,3,..., m-n	i	anything	e	$P_{n,n+i} = Q_a(i,n)$

Solve for  $p_n$ , then  $N_{\text{bar}} = \text{Expect \# backlogged packets then use}$   
 Little's law:  $T = \lambda/2$  to get mean delay.

( $N_{\text{bar}}$  near 0 means almost no delay to xmit)

[Recall: will be lower bound on delay, because we assume unbuffered system]

Something is missing in analysis:

Dynamics of system

small  $q_r \Rightarrow$  large delay

large  $q_r \Rightarrow$  n grows and thus delay grows (more collisions)

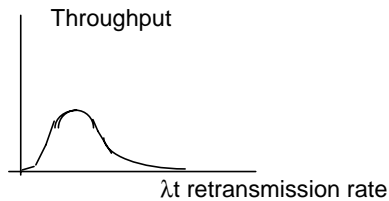
The problem is that "optimal"  $q_r$  is function of n, but n is a result of  $q_r$ .

$\Rightarrow$  we want to explore this **feedback** with our model.

n small  $\Rightarrow$  make  $q_r$  large

n large  $\Rightarrow$  make  $q_r$  small

Let's calculate the departure rate, so we can graph:



$$\begin{aligned}\text{Departure rate} &= E[\text{successful xmits in one slot}] \\ &= \Pr[\text{one node successfully transmits}] \\ &= P_{\text{succ}}\end{aligned}$$

$$\begin{aligned}P_{\text{succ}} &= (\text{see table, for "1" result}) \\ &= P[0 \text{ arrive, 1 trans}] + P[1 \text{ arrive, 0 retrans}] \\ &= Q_a(0, n) Q_r(1, n) + Q_a(1, n) Q_r(0, n)\end{aligned}$$

If we substitute our expressions for  $Q_a$  and  $Q_r$ , and use the approximation  $(1-x)^y = e^{-xy}$  for small  $x$ , then:

$$P_{\text{succ}} = G(n)e^{-G(n)}$$

$$\begin{aligned}G(n) &= E[\text{\# attempted xmits in a slot when system in state } n] \\ &= (m-n)q_a + nq_r \\ &= \text{arrival rate} + \text{retransmission rate}\end{aligned}$$

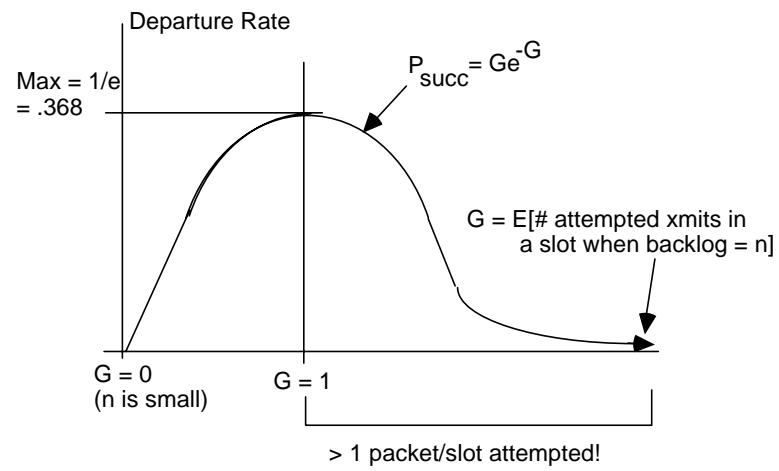
This is the same as  $\Pr[1 \text{ arrival}]$  in  $\tau = 1$  interval in Poisson process with parameter  $G(n)\tau$ : ( $\tau = 1$  interval is one slot)

$$\Pr[n \text{ departures/arrivals}] = e^{-G(n)\tau} (G(n)\tau)^n / n!, \text{ with } n = 1, \tau = 1$$

$$= G(n) e^{-G(n)}$$

Approximate # departures in slot as Poisson process with parameter  $G(n)$ , which varies with system state.

Now we can look at Throughput:  $T_{\text{put}} = \text{Departure rate} \cdot .368$



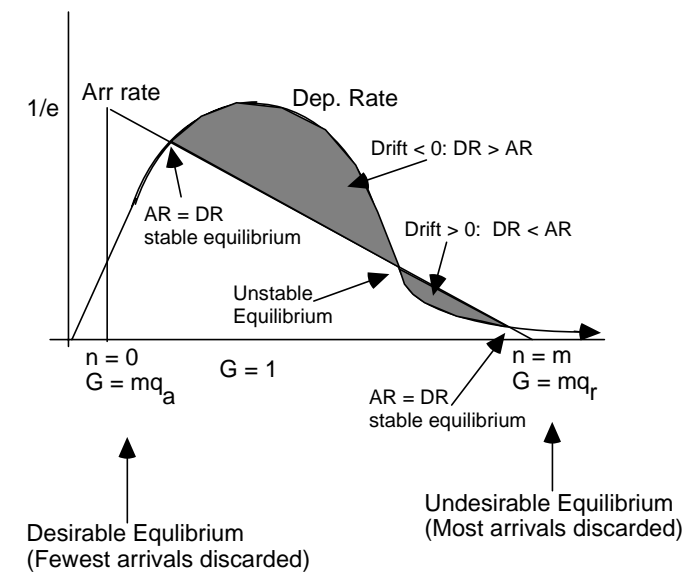
Same behavior we discussed in Internet: # collisions grows, driving down departure rate.  
 $\Rightarrow$  Arrivals are lost.

Now we have a graph of departure rate vs.  $G(n)$ . But how is arrival rate related to departure rate?

Let's superimpose a graph of arrival rate:

$$E[\text{arrival rate}] = (m-n)q_a$$

Note that x axis is  $n$ , as well as  $G(n)$



We will define DRIFT as the shaded region:

Drift in state  $n$  ( $D_n$ ) = expected change in backlog over one slot time.

$$D_n = E[\text{arrival rate}] - E[\text{departure rate}]$$

Using the expression from above:

$$D_n = (m-n)q_a - P_{\text{succ}}$$

$$P_{\text{succ}} = \Pr[\text{successful xmit at a node}]$$

$D_n > 1$ :  $AR > DR \rightarrow n$  will increase

$D_n < 1$ :  $DR > AR \rightarrow n$  will decrease

Increase  $q_r$ : If scale for  $n$  is fixed, causes horizontal contraction of  $G(n)$  scale.  
 $\Rightarrow$  Unstable equilibrium occurs at smaller  $n$ !

Decrease  $q_r$ : Unstable equilibrium occurs at larger  $n$  (but delay also increases) & lots of arrivals discarded. As you decrease  $q_r$ , eventually have only 1 (desirable) stable point.

### Summary of 4.2, BG

- Want delay/tput as function of offered load
- 6 assumptions
- Consider no buffering
- Model as discrete time Markov chain:  
State = # backlogged nodes
- Model gives  $E[\text{\#backlogged nodes}] = N_{\text{bar}}$ , Little's law for  $T(\text{delay})$
- Steady state chain solution does not reveal "equilibrium" point.
- Also, model is based on fixed  $q_r$ . In reality,  $q_r$  may vary.  
small  $n \Rightarrow$  increase  $q_r$   
large  $n \Rightarrow$  decrease  $q_r$

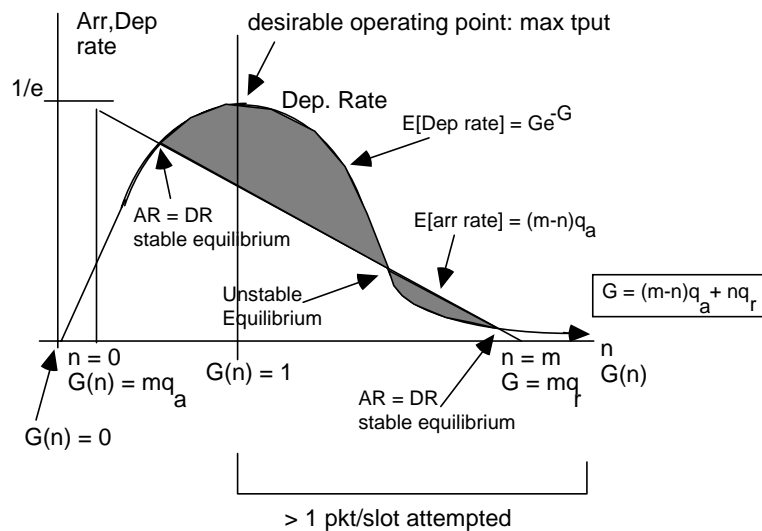
How can we capture this feedback dynamic relationship?

$$\text{Drift} = D_n = E[\text{Arr rate}] - E[\text{depart. rate}]$$

$$= (m-n)q_a = G(n)e^{-G(n)}$$

$(m-n)q_a$  = poisson arrival with rate  $\lambda$

$G(n)e^{-G(n)}$  = Approx. depart. process as Poisson w/parm  $G(n)$   
 $\Rightarrow$  this is  $\Pr[1 \text{ depart}]$  w/ $\tau = 1$  using Poisson distribution (assumes  $\lambda$  small (« 36.8%))  
 $G$  is "attempt rate"



Now consider infinite node assumption:

attempted departure rate (including collisions) =

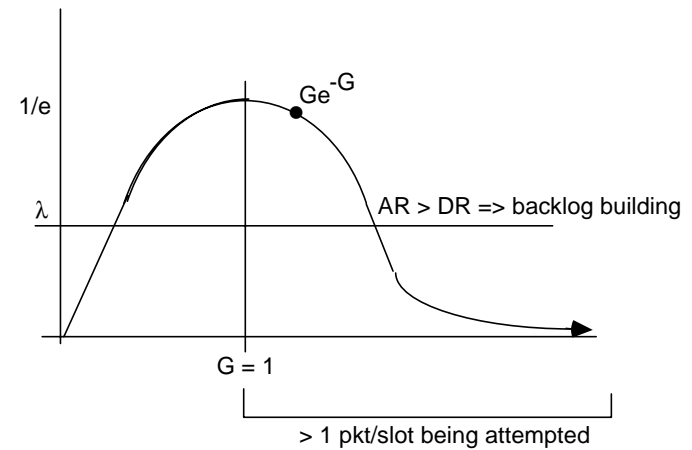
$$G(n) = \lambda + n q_r$$

**Arrival rate curve is horizontal line (constant  $\lambda$ )**

Markovian chain has no state distribution

Expected backlog increases w/o bound!

=> Expected delay is  $\infty$  for any  $\lambda$ .





Practical considerations:

- 1) if not slotted (var length msgs), max tput drops in half
- 2) We'll see, later, way to get close to 50% tput.
- 3) In practice, if  $\lambda \ll 1/e$  and  $q_r$  "moderate" (not too small)  
System may run for long time w/o entering unstable state (remain there for long time)

Then restart system to recover.

However, better solutions exist:

BG 4.2.3 Stabilized Slotted Aloha => change  $q_r$  dynamically

$P_{succ}$  has max at  $G(n) = 1$ .

Problem: Nodes don't know what  $n$  is; must estimate from feedback.

Example: increase  $q_r$  in idle slot  
decrease  $q_r$  in e slot

Definition:

multiaccess system is stable if, given  $\lambda$ ,  $E[\text{delay}/\text{pkt}]$  is finite.

Definition: Max stable tput: least upper bound of arrival rates for which system is stable

Ordinary SA is unstable: Max stable TPUT = 0 (with 6b (• buffer  
Stabilized SA is stable: Max stable TPUT =  $1/e$  assumption)

p286-287

Ethernet differs from above: limited feedback

Each node has feedback only when it xmits. Violates earlier assumption. (Other differences - carrier sense)

Binary Exponential Backoff

- xmit arrival immediately
- If xmit unsuccessful  $i$  times, then probability of retransmit in successive slots is  $1/2^i$

Max stable TPUT  $\rightarrow 0$  as  $m \rightarrow \infty$ , however!

Open Problem: Does any strategy exist that is stable, with this limited feedback?

Q: Do you think that Ethernets really are limited to 36.8% x 10Mbit/sec?

A: No! We saw that in graphs from [AC] paper!

Q: What's going on then? How was Ethernet invented?

A: Metcalfe was reading Proceedings of Fall Joint Computer Conference 1970:

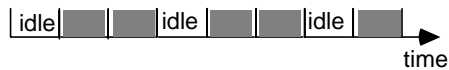
The Aloha System - another alt. for computer comm, to sleep.

Redid analysis in 1973, not sure if he ever finished at Harvard; left for PARC.

Published Ethernet paper in 1976 with Boggs

#### BG 4.4 Carrier Sensing

Recall that slotted ALOHA has idle & busy slots of equal length.

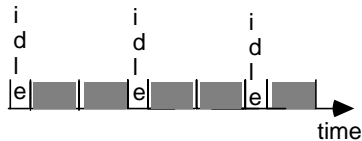


Why waste a full slot for IDLE?

If there (max prop delay + detection delay)  $\ll 1$

(where max prop delay + detection delay has  $\beta$  as an upper bound)

Then we can terminate idle slots after time  $\beta < 1$



The process of detecting idle slots is like detecting (sensing) an (imaginary) carrier.

Hence the name: CSMA (carrier sense, multiple access)

#### BG 4.4.1 CSMA Slotted Aloha

Two differences from ordinary slotted aloha:

- 1) Terminate idle periods after time  $\beta$
- 2) If packet arrives at a node while xmit is in progress, the packet is regarded as backlogged. Several variations of when to transmit this packet:

A) Non-Persistent CSMA:

- i) xmit packet arriving in idle slot in next slot as usual
- ii) xmit packet arriving in busy slot w/prob  $q_r$  after each subsequent idle slot

(better utilization than 1-persistent, due to lower prob. of collision for arrivals; but higher delay than 1-persistent)

B) P-Persistent CSMA:

- i) (same rule)
- ii) use probability  $p$ , not  $q_r$ , for 1st try. Use  $q_r$  for subsequent tries.

1-Persistent CSMA is special case of P-Persistent. Only difference from ordinary slotted aloha is carrier sense.

(better performance than ordinary slotted aloha due to CS mechanism).

#### BG 4.1 CSMA slotted Aloha

##### - Modelling

two slot durations:  $\begin{cases} \beta = .01 - .10 \\ 1 \end{cases}$

**Idle slot always terminate after  $\beta$  time units and a new slot begins.**

Model of BG4.2 can be modified. (No longer transitions at discrete time)

Modify drift time to derive attempted departure:

rate(BG fig 4.16): (assumption 6b ( $\bullet$  nodes))

$$D = ge^{-g}/(\beta + 1 - e^{-g})$$

$$w/\max \text{ of } 1/(1 + \bullet(2\beta))$$

$$\text{and } g(n) = \lambda\beta + q_n r_n$$

Important result:

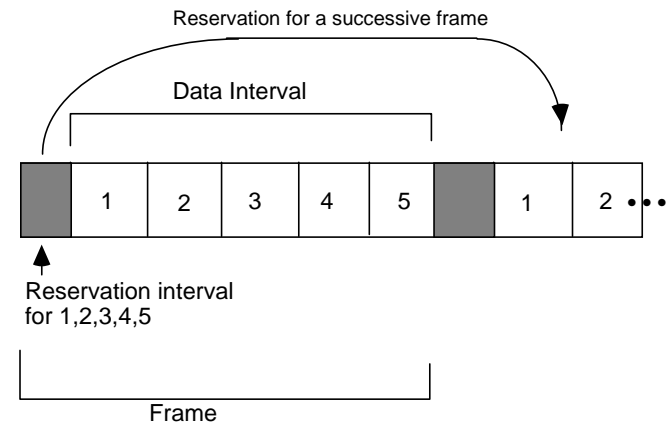
$\lim_{\beta \rightarrow 0} D = 1 \Rightarrow \text{near } 100\% \text{ tput}$

See figure BG 4.16, Tanenbaum3.5

#### BG 4.5 Multiaccess Reservations

General idea:

if  $\beta$  large (as in a satellite), let stations send tiny packets in contention mode or TDM to reserve future slots.  
 $\Rightarrow$  higher overall efficiency.



Using reservations w/SA yields 36% max utilization for reservation interval, 88% for entire BW  
(assumes reserve slot is 5% of data slot)

[BG 4.5.2] Ethernet = CSMA/CD

Collision Detect added to CSMA at start of slot:  
one reservation slot.

$S > 1/(1 + 6.2\beta)$  (vs.  $1/(1 + 1.4\beta)$  for no CD w/non persistent)  
(for  $\beta = 1/100$ , CSMA/CD = 94, CSMA = 87)

[p BG 320] IEEE 802 standards

IEEE 802.3 (Ethernet) is:  
- CSMA/CD

- binary exp. backoff
- unslotted persistent

[Stallings 3rd ed, p 425] see figure 11.23

#### BG 4.5.3 Token Rings

- IBM - Zurich lab developed during Ethernet standardization process
- Better delay characteristics under heavy load than CSMA/CD
- A conservative protocol from a conservative company!

(See fig. 1 [Hawe,Kempf,Kirby - DigTech Journal, Sept 86])

(See fig 3.20 [Tanenbaum])

(See fig 11-15 [Stallings])

(See fig 3-26, 3-27 [Tanenbaum])

(See fig 3-21 [Tanenbaum])