

**Lecture 11 – part 2**  
**CS/EE 5516 - Spring 1994**  
**Performance Modeling**

**References:**

[BG] Chapter 3: 3.1, 3.2.1, 3.2.3, 3.3.1, 3.3.2, ...  
C. H. Sauer and K. M. Chandy, *Computer Systems Performance Modeling*, 2.4.2, 3.2.1-3.2.2, 5

Topics:

1. Performance measures of interest
2. Little's Law and its application
3. Poisson process and the exponential distribution
4. Memoryless property
5. Markovian queue
6. Non-Markovian Queues
7. Networks of Markovian queues

Topic 6: Non-Markovian Queues ([BG] 3.1)

Notation:

- $M/M/1$  Exponential interarrival and interservice times; 1 server
- $M/M/n$  Not 1 but n servers
- $M/M/1/k$  Queue only has room for k customers; others lost
- $M/G/1$  General interservice time distribution

Solved by using Markov chain that exists at the instance a service departure occurs; this is called an *embedded Markov chain*

- $G/M/1$  General interarrival time distribution
- $G/G/1$  General interarrival and interservice distributions

Topic 7: Networks of Markovian Queues

General comments:

- Queueing discipline at each queue must be specified (e.g., First-come, first-served)
- Forks in the network are usually assigned probabilities

- There may be multiple “classes” of customers
- “Flows” must match: flow out of a server must equal flow into all downstream servers that the output is connected to.

What is a “solution” to a queueing network?

- Finding number of customers at queue  $i$

Two type of networks:

- Open
  - Good to model datagram network
  - One or more external sources of customers
  - One or more external sinks to which customers depart
  - Queues have unbounded length
  - Solve by computing “relative throughputs” starting at source nodes.
- Closed
  - Good to model sliding window
  - Population (number of customers) is fixed
  - No sources or sinks
  - Max customers at any queue is bounded above by network population
  - Solve by drawing a chain. State in each node is an k-tuple, where k is number of servers in network:  $(n_0, n_1, \dots, n_k)$
  - Key result: product form solution:  
Probability( $n_0$  customers at queue 0,  $n_1$  at queue 1, ...) =  
 $\Pr(n_0 \text{ at queue 0}) * \Pr(n_1 \text{ at queue 1}) * \dots$

