Homework 6

CS 5114 (Spring 2013)

Assigned on April 9, 2013.
Submit PDF solutions by email to the TA
beginning of class on April 16, 2013.

Instructions:

- You are not allowed to consult any sources other than your textbook, the slides on the course web page, your own class notes, the TA, and the instructor. In particular, do not use a search engine.
- Do not forget to typeset your solutions. *Every mathematical expression must be typeset as a mathematical expression, e.g., the square of n must appear as n^2 and not as "n^2".* Students can use the LaTeX version of the homework problems to start entering their solutions.
- Describe your algorithms as clearly as possible. The style used in the book is fine, as long as your description is not ambiguous. Explain your algorithm in words. A step-wise description is fine. *However, if you submit detailed pseudo-code without an explanation, we will not grade your solutions.*
- Do not make any assumptions not stated in the problem. If you do make any assumptions, state them clearly, and explain why the assumption does not decrease the generality of your solution.
- Do not describe your algorithms only for a specific example you may have worked out.
- You must also provide a clear proof that your solution is correct (or a counter-example, where applicable). Type out all the statements you need to complete your proof. *You must convince us that you can write out the complete proof. You will lose points if you work out some details of the proof in your head but do not type them out in your solution.*
- Describe an analysis of your algorithm and state and prove the running time. You will only get partial credit if your analysis is not tight, i.e., if the bound you prove for your algorithm is not the best upper bound possible.

Problem 1 (10 points) Solve exercise 4 in Chapter 7 (page 416) of your textbook.

Problem 2 (10 points) Solve exercise 5 in Chapter 7 (page 416) of your textbook.

Problem 3 (20 points) Solve exercise 6 in Chapter 7 (pages 416–417) of your textbook.

Problem 4 (30 points) Solve exercise 22 in Chapter 7 (pages 428) of your textbook.

Problem 5 (30 points) A flow network has m edges with integer capacities. Show that there exists a sequence of at most m augmenting paths that yield the maximum flow. In other words, if you prove this statement, you would have shown that if the augmenting paths were chosen correctly (by magic), the Ford-Fulkerson algorithm will terminate in at most m iterations. Note that this question is not algorithmic, i.e., you do not have to demonstrate an algorithm to compute these augmenting paths. You only have to prove the existence of these augmenting paths. (*Hint:* Suppose that you knew the maximum flow in the network. How could you discover these m magical augmenting paths?)