Some NP-Complete Problems

SATISFIABILITY(SAT)
INSTANCE: A Boolean expression E over variables $x_1, x_2, \ldots, x_n$ in conjunctive normal form.
QUESTION: Is there an assignment of truth values to $x_1, x_2, \ldots, x_n$ making E true?

3-SAT
INSTANCE: A Boolean expression E in conjunctive normal form such that each clause contains exactly 3 literals.
QUESTION: Is there a satisfying assignment for E?

3-COLORABILITY
INSTANCE: Graph $G = (V, E)$.
QUESTION: Is G 3-colorable, that is, is there a function $f : V \rightarrow \{\text{red}, \text{blue}, \text{green}\}$ such that $f(u) \neq f(v)$ whenever $(u, v) \in E$?

3-DIMENSIONAL MATCHING (3DM)
INSTANCE: A set $M \subset W \times X \times Y$ where W, X, and Y are disjoint sets having the same number q of elements.
QUESTION: Does M contain a matching, i.e., a subset $M' \subset M$ such that $|M'| = q$ and no two elements of M' agree in any coordinate?

EXACT COVER BY 3-SETS (X3C)
INSTANCE: Finite set X with $|X| = 3q$, q an integer; collection C of 3-element subset of X.
QUESTION: Does C contain an exact cover for X, i.e., a subcollection $C' \subset C$ such that every element of X occurs in exactly one member of C'?

PARTITION
INSTANCE: A finite set A, and a "size" $s(a) \geq 0$ defined for each $a \in A$.
QUESTION: Is there a subset $A' \subset A$ such that

$$\sum_{a \in A'} s(a) = \sum_{a \in A - A'} s(a)?$$

KNAPSACK
INSTANCE: Items $1, \ldots, N$ with $size(i) \geq 0$ and $value(i) \geq 0$ defined for each item i; integers $M, K \geq 0$.
QUESTION: Is there a subset $S \subset \{1, \ldots, N\}$ such that

$$\sum_{i \in S} size(i) \leq M$$

and

$$\sum_{i \in S} value(i) \geq K?$$

CLIQUE
INSTANCE: Undirected graph $G = (V, E)$, positive integer $K \leq |V|$.
QUESTION: Does G have a clique of size K or more, i.e., a subset $V' \subset V$ with $|V'| \geq K$ such that every two vertices of V' are adjacent?

INDEPENDENT SET
INSTANCE: Undirected graph $G = (V, E)$; positive integer $K \leq |V|$.
QUESTION: Does G contain an independent set of size K or more, i.e., a subset $V' \subset V$ such that $|V'| \geq K$ and such that no two vertices of V' are adjacent?

VERTEX COVER (VC)
INSTANCE: Undirected graph $G = (V, E)$; positive integer $K \leq |V|$.
QUESTION: Is there a vertex cover of size K or less for G, i.e., a subset $V' \subset V$ such that $|V'| \leq K$ and such that for each $(u, v) \in E$, either $u \in V'$ or $v \in V'$?

DOMINATING SET
INSTANCE: Undirected graph $G = (V, E)$; positive integer $K \leq |V|$.
QUESTION: Does G contain a dominating set of size K or less, i.e., a subset $V' \subset V$ with $|V'| \leq K$ such that for all $u \in V - V'$ there is a $v \in V'$ for which $(u, v) \in E$?

HAMILTONIAN CIRCUIT (HC)
INSTANCE: Undirected graph $G = (V, E)$.
QUESTION: Does G contain a Hamiltonian circuit, i.e., a simple cycle of length $|V|$?

End of List