CS 5114
Homework Exercise 6

Given: March 7, 2000 **Due:** March 31, 2000

The point value of each problem is shown in [ ]. Each solution must include all calculations and an explanation of why the given solution is correct. In particular, write complete sentences. A correct answer without an explanation is worth no credit. The assignment must be *submitted* to the instructor by 12:00 noon on March 31, 2000. See syllabus for late policy.

Electronic preparation of your solutions in LaTeX is mandatory. Here is the suggested procedure.

> Retrieve this LaTeX source file `homework6.tex` from the 5114 Web pages and rename it `solvehw6.tex`. Delete these instructions. Enter your solutions in the locations explained by LaTeX comments (`%`). Also enter your name in the `\student` command and uncomment the line near the beginning of the file that uses the `\student` command. When you are satisfied with your solutions, print a copy and turn it in during class or no later than noon on March 31, 2000.

Electronic submission is optional. If you use electronic submission, send an email to `cs5114@courses.cs.vt.edu` with subject `Solutions to Homework Assignment 6` and with two attachments: `solvehw6.tex` and `solvehw6.ps`. Your email must be *received* by 12:00 noon on March 31, 2000.

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[15] 1. Let C be the set of composite integers

$$C = \{10, 26, 39, 51, 54, 55, 68, 81, 91, 93\}.$$

Let P be the set of prime factors of integers in C. Define an undirected graph $G = (V, E)$ by

$$V = C \cup P,$$

and

$$E = \{(m, p) : m \in C \text{ and } p \text{ divides } m\}.$$

Use network flow techniques to find a maximum matching in G. What network do you use, what is the max flow you find, and how do you determine a maximum matching?

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[15] 2. A string of characters $S = a_1 a_2 \cdots a_n$ can sometimes be expressed more briefly in terms of one of its substrings. For example, some strings are powers:

$$\begin{aligned} aaaaaaaaa &= a^9 \\ 373737373737 &= (37)^6. \end{aligned}$$

To generalize this, we can define $\text{root}(S)$, the *root of the string* S, to be the shortest string R such that S is a prefix of some power of R. We get

$$\begin{aligned} \text{root}(aaaaaaaaa) &= a \\ \text{root}(373737373737) &= 37 \\ \text{root}(ababa) &= ab \\ \text{root}(4127412741274) &= 4127. \end{aligned}$$

Of course, there are cases in which $\text{root}(S) = S$:

$$\begin{aligned} \text{root}(bccdbcd) &= bccdbcd \\ \text{root}(XXXY) &= XXXY. \end{aligned}$$

Give pseudocode for an algorithm EXTRACT-ROOT with optimal time complexity to find the root of a string S of length n. What is the time complexity of your algorithm?

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[15] 3. CLR Exercise 35.1-4.

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[15] 4. CLR Problem 35-3. When you see the $O(n \lg n)$ in part **a,** you should think "sorting." Try some small examples to get some intuition on how you can pick out an appropriate Ghostbuster/ghost pair.