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CS 4604: Introduction to Database Management Systems

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Lecture #13: Functional Dependencies

Course Outline

- Weeks 1–4: Query/ Manipulation Languages and Data Modeling
  - Relational Algebra
  - Data definition
  - Programming with SQL
  - Entity-Relationship (E/R) approach
  - Specifying Constraints
  - Good E/R design
- Weeks 5–8: Indexes, Processing and Optimization
  - Storing
  - Hashing/Sorting
  - Query Optimization
  - NoSQL and Hadoop
- Week 9-10: Relational Design
  - Functional Dependencies
  - Normalization to avoid redundancy
- Week 11-12: Concurrency Control
  - Transactions
  - Logging and Recovery
- Week 13–14: Students' choice
  - Practice Problems
  - XML
  - Data mining and warehousing

Announcements

- Handout 3 on FDs and Normalization is out.
  - We will discuss it next Tue, Oct 23

Functional Dependencies and Schema Normalization

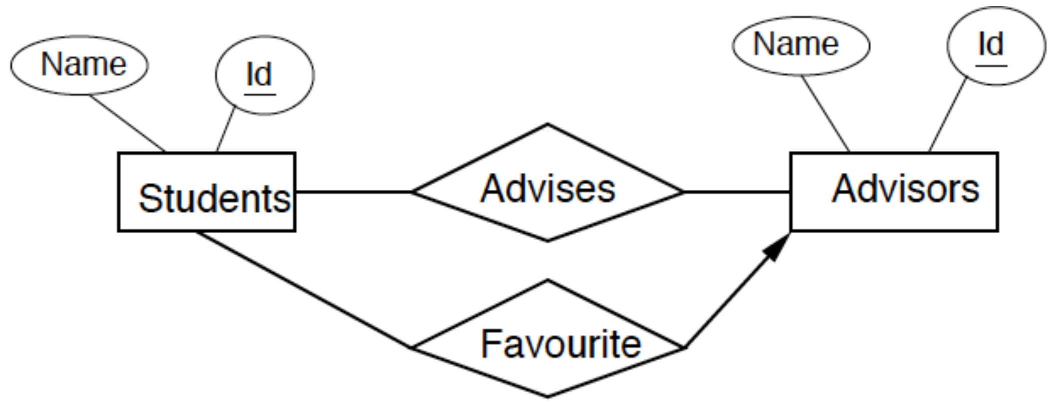
- A bit abstract and theoretical!
- But important!

- Plan: 3 lectures
  - – 1. What are FDs? How to reason about them?
  - – 2. BCNF, 3NF and Normalization
  - – 3. Practice Problems in class

Overview

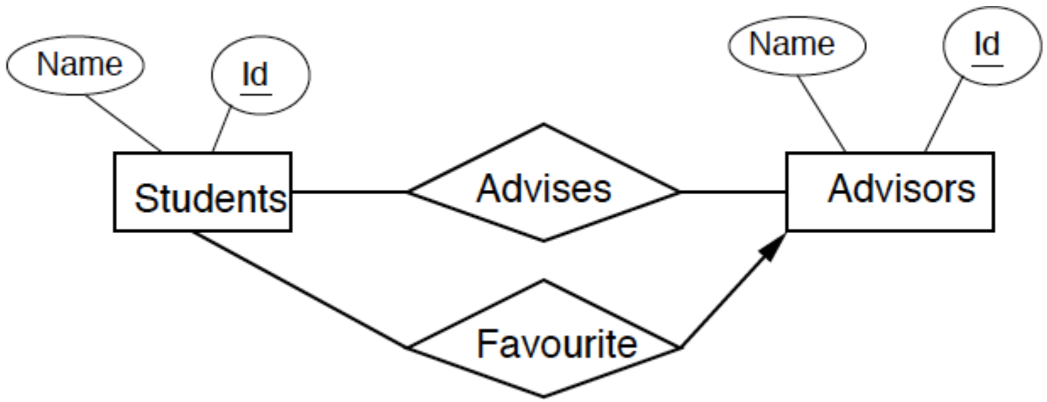
- Functional dependencies
  - **why**
  - Definition
  - Attribute closures and keys
  - Armstrong's "axioms"
  - FD closure and cover

Example



- Convert to relations
  - Students (ID, Name)
  - Advisors (ID, Name)
  - Favourite (StudentID, AdvisorID)
  - Advises (StudentID, AdvisorID)

Example



- What if we combine Students, Advises, and Favourite into one relation?
  - Students(Id, Name, AdvisorId, AdvisorName, FavouriteAdvisorId)
  - Seems ‘intuitively bad’ right?

Example of a Bad Relation

Students(Id, Name, AdvisorId, AdvisorName, FavouriteAdvisorId)

- What makes it bad?
- Given the Student's Id, can any other values be determined?
  - Name and FavouriteAdvisorId
  - Id → Name
  - Id → FavouriteAdvisorId

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Example of a Bad Relation

Students(Id, Name, AdvisorId, AdvisorName, FavouriteAdvisorId)

- Name and FavouriteAdvisorId
- Id $\rightarrow$ Name
- Id $\rightarrow$ FavouriteAdvisorId
- AdvisorId $\rightarrow$?

Example of a Bad Relation

Students(Id, Name, AdvisorId, AdvisorName, FavouriteAdvisorId)

- Name and FavouriteAdvisorId
- Id → Name
- Id → FavouriteAdvisorId
- AdvisorId → AdvisorName

- Can we say Id → AdvisorId ?
  - Not really! Why?
  - Id is a not a key for Students relation
  - Key: {Id, AdvisorId}

Example of a Bad Relation

Students(Id, Name, AdvisorId, AdvisorName, FavouriteAdvisorId)

- OK, what ***really*** makes it bad?
- Ans: Parts of the key determine other attributes
- Leads to:
  - Redundancy (Space, Inconsistencies, ...)

Motivation for Functional Dependencies

- Reason about constraints on attributes in a relation
- Procedurally determine the keys of a relation
- Detect when a relation has redundant information
- Improve database designs systematically using normalization

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Definition of FD (Functional Dependency)

- X → Y

'X' functionally determines 'Y'

Informally: 'if you know 'X', there is only one 'Y' to match'

Definition of FD (Functional Dependency)

- ((If t is a tuple in a relation R and A is an attribute of R, then t[A] is the value of attribute A in tuple t))

- Formally:

$X \rightarrow Y \Rightarrow (t1[X] = t2[X] \Rightarrow t1[Y] = t2[Y])$

if two tuples agree on the 'X' attribute, they *must* agree on the 'Y' attribute, too

(eg., if ids are the same, so should be names)

$X \rightarrow Y$

- X and Y can be **sets** of attributes

- Definition of FDs
- A FD on a relation R is a statement:
  - If two tuples in R agree on attributes A1, A2, …, An they agree on attribute B
  - Notation: A1 A2 … An → B

Definitions contd.

- A FD is a **constraint** on a single relational **schema**
  - It must hold on every **instance** of the relation
  - **You can not deduce an FD from a relation instance!**
  - (but you can deduce if an FD does NOT hold using an instance)

Examples of FDs

- List the FDs

Courses(Number, DeptName, CourseName, Classroom, Enrollment)

Number	DeptName	CourseName	Classroom	Enrollment
4604	CS	Databases	TORG 1020	45
4604	Dance	Tree Dancing	Drillfield	45
4604	English	The Basis of Data	Williams 44	45
2604	CS	Data Structures	MCB 114	100
2604	Physics	Dark Matter	Williams 44	100

- Number DeptName → CourseName
- Number DeptName → Classroom
- Number DeptName → Enrollment
- Number DeptName → CourseName Classroom Enrollment

Examples of FDs

- List the FDs

`Courses(Number, DeptName, CourseName, Classroom, Enrollment)`

Number	DeptName	CourseName	Classroom	Enrollment
4604	CS	Databases	TORG 1020	45
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2604	Physics	Dark Matter	Williams 44	100

- Is Number → Enrollment an FD?

Where do FDs come from?

- "Keyness" of attributes
- Domain and application constraints
- Real world constraints
  - E.g. ProfessorID Time → Classroom

Definition of Keys

- FDs allow us to formally define keys
- A set of attributes {A1, A2, …, An} is a key for relation R if:

Uniqueness: {A1, A2, …, An} functionally determine all the other attributes of R

Minimality: no proper set of {A1, A2, …, An} functionally determines all other attributes of R.

Definitions of Keys

- A superkey is a set of attributes that has the uniqueness property but is not necessarily minimal
- If a relation has multiple keys, specify one to be primary key
- Convention: underline the attributes (but you know that!)
- If a key has only one attribute A, say A rather than {A}

Example of keys

- What is the key for

Courses (Number, DeptName, CourseName, Classroom, Enrollment) ?

- The key is {Number, DeptName}
  - These attributes functionally determine every other attribute
  - No proper subset of {Number, DeptName} has this property

Example of Keys

- What is the key for

Teach (Number, DepartmentName, ProfessorName, Classroom) ?

- The key is {Number, DepartmentName}
  - Why?

Keys in E/R to Relational Conversion

- From an ENTITY SET

If the relation comes from an entity set, the key attributes of the relation are precisely the key attributes of the entity set

Keys in E/R to Relational Conversion

- From a RELATIONSHIP (binary for now between E and F)
- R is many-many:
  - Key attributes of the relation are the key attributes of E and of F
- R is many-one:
  - Key attributes of the relation are the key attributes of E
- R is one-one:
  - Key attributes of the relation are the key attributes of E or of F

Keys in E/R to Relational Conversion

- From a RELATIONSHIP (multiway?)
- Need to reason about the FDs that R satisfies
- No simple rule
- If R has an arrow towards entity set E, at least one key for the relation for R excludes the key for E

Rules for Manipulating FDs

- Learn how to reason about FDs
- Define rules for deriving new FDs from a given set of FDs
- Example: R (A, B, C) satisfies FDs $A \rightarrow B$, $B \rightarrow C$.
  - What others does it satisfy?
  - $A \rightarrow C$
  - What is the key for R?
  - A (as $A \rightarrow B$ and $A \rightarrow C$)

Equivalence of FDs

- Why?
  - To derive new FDs from a set of FDs
- An FD F follows from a set of FDs T if every relation instance that satisfies all the FDs in T also satisfies F
  - $A \rightarrow C$ follows from $T = \{A \rightarrow B, B \rightarrow C\}$
- Two sets of FDs S and T are equivalent if each FD in S follows from T and each FD in T follows from S
  - $S = \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$ and $T = \{A \rightarrow B, B \rightarrow C\}$ are equivalent

Splitting and Combining FDs

- The set of FDs
  - A1 A2 A3…An → B1
  - A1 A2 A3…An → B2
  - …

  is equivalent to the FD
  - A1 A2 A3…An → B1 B2 B3 … Bm
- This equivalence implies two rules:
  - Splitting rule
  - Combining rule
  - These rules work because all the FDs in S and T have identical left hand sides

Splitting and Combining FDs

- Can we split and combine left hand sides of FDs?
- For the relation Courses, is the FD
  - Number DeptName $\rightarrow$ CourseName

  equivalent to the set of FDs
  - {Number $\rightarrow$ CourseName, DeptName $\rightarrow$ CourseName} ?
  - NO

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Triviality of FDs

- A FD A1 A2…An → B1 B2…Bm is
  - Trivial if the B's are a subset of the A's

    $$\{B_1, B_2, \ldots B_n\} \subseteq \{A_1, A_2, \ldots A_n\}$$

  - Non-trivial if at least one B is not among the A's

    $$\{B_1, B_2, \ldots B_n\} - \{A_1, A_2, \ldots A_n\} \neq \emptyset$$

  - Completely non-trivial if none of the B's are among the A's

    $$\{B_1, B_2, \ldots B_n\} \cap \{A_1, A_2, \ldots A_n\} = \emptyset$$

Triviality of FDs

- What good are trivial and non-trivial FDs?
  - Trivial dependencies are always true
  - They help simplify reasoning about FDs
- Trivial dependency rule: The FD A1 A2…An → B1 B2…Bm is equivalent to the FD A1 A2…An → C1 C2..Ck, where the C's are those B's that are not A's i.e.

$$\{C_1, C_2, \ldots, C_k\} = \{B_1, B_2, \ldots, B_m\} - \{A_1, A_2, \ldots, A_n\}$$

Overview

- Functional dependencies
  - why
  - Definition
  - **Attribute closures and keys**
  - Armstrong's "axioms"
  - FD closure and cover

Closure of Attributes: Example

- Suppose a relation R (A, B, C, D, E, F) has FDs:
  - AB → C, BC → AD, D → E, CF → B

- Question:

Find set X of attributes such that AB → X is true

- Answer:

X = {A, B, C, D, E} i.e. AB → ABCDE

Closure of Attributes: Example

- Suppose a relation R (A, B, C, D, E, F) has FDs:
  - AB → C, BC → AD, D → E, CF → B

What is BCF ?

- Question:

Find set Y of attributes such that BCF → Y is true

ans: A superkey

- Answer:

Y = {A, B, C, D, E, F} i.e. BCF → ABCDEF

Closure of Attributes: Example

- Suppose a relation R (A, B, C, D, E, F) has FDs:
  - AB → C, BC → AD, D → E, CF → B

- Question:

Find set Z of attributes such that AF → Z is true

- Answer:

Y = {A, F} i.e. AF → AF

Closure of Attributes: Example

- Suppose a relation R (A, B, C, D, E, F) has FDs:
  - $AB \rightarrow C$, $BC \rightarrow AD$, $D \rightarrow E$, $CF \rightarrow B$

- X, Y, Z are the **closures** of {A, B}, {B, C, F}, and {A, F} respectively

Attribute Closure, another way of looking (not in book)

R(A, B, C)

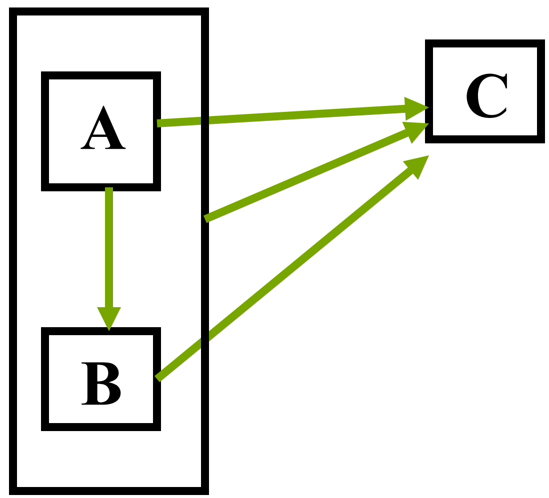
FD set:

AB->C (1)

A->BC (2)

B->C (3)

A->B (4)



Closure of Attributes: Definition

- Given:
  - Attributes {A1, A2, …, An}
  - A set of FDs S
- The **Closure** of {A1, A2, …, An} under S is
  - the set of attributes {B1, B2, …, Bm} such that for $1 <= i <= m$, the FD A1 A2 … An $\rightarrow$ Bi follows from S
  - the closure is denoted by $\{A1, A2, \ldots, An\}^+$

Closure of Attributes: Definition

- Question:

Which attributes must $\{A1, A2, \ldots, An\}^+$ contain at the minimum?

- Answer:

$\{A1, A2, \ldots, An\}$

- Why?

$A1\ A2 \ldots An \rightarrow Ai$ is a trivial FD

Closure of Attributes: Algorithm

- Given (INPUT) :
  - Attributes {A1, A2, .. An}
  - Set of FDs S

- Find (OUTPUT) :
  - $X = \{A1, A2, \ldots, An\}^{+}$

Closure of Attributes: Algorithm

1. Use the splitting rule so that each FD in S has one attribute on the right.
2. Set X == {A1, A2 ..., An}
3. Find FD B1 B2…Bk → C in S such that

{B1 B2 … Bk} $\subseteq$ X but C $\notin$ X

4. Add C to X
5. Repeat the last two steps until you can't find C

Why is the algorithm correct?

Why compute Attribute Closures?

- Prove correctness of rules for manipulating FDs

Example:

Prove the transitive rule i.e.

IF

A1 A2 … An → B1 B2 … Bm

B1 B2 … Bm → C1 C2 … Ck

THEN

A1 A2 … An → C1 C2 … Ck

To prove this, check if

$$\{C_1, C_2, \ldots, C_k\} \subseteq \{A_1, A_2, \ldots, A_n\}^+$$

Why compute Attribute closures?

- Check if a "new" FD A1, A2, … An $\rightarrow$ B follows from a set of FDs S

Simply check if B is in $\{A1, A2, \ldots, An\}^+$ under S

- Get keys procedurally (aka algorithmically)

A set of attributes X is a key for a relation R iff

  – $\{X\}^+$ is the set of all attributes of R
  – For no attribute $A \in X$ is $\{X - \{A\}\}^+$ the set of all attributes of R

Examples of Closure Computations

- Consider the “bad” relation

Students (Id, Name, AdvisorId, AdvisorName, FavouriteAdvisorId)

- What are the FDs that hold in this relation?
  - Id → Name
  - Id → FavouriteAdvisorId
  - AdvisorId → AdvisorName
- To compute the key for this relation:
  - Compute the closures for all sets of attributes
  - Find the minimal set of attributes whose closure is the set of all attributes

Algorithm for computing keys

- Given (INPUT) :
  - A relation R (A1, A2, …, An)
  - The set of all FDs S that hold in R
- Find (OUTPUT) :
  - Compute all the keys of R

1. For every subset K of {A1, A2, …, An} compute its closure
2. If {K}+ = {A1, A2, … An} and for every attribute A, {K – {A}}+ is not {A1, A2, … An}, then output K as a key

- Running time?

Overview

- Functional dependencies
  - why
  - Definition
  - Attribute closures and keys
  - **Armstrong’s “axioms”**
  - FD closure and cover

Armstrong's Axioms

- We can use closures of attributes to determine if any FD follows from a given set of FDs

OR

- Use Armstrong's axioms: complete set of inference rules from which it is possible to derive every FD that follows from a given set:

Armstrong's Axioms

- **Reflexivity**

$$Y \subseteq X \Rightarrow X \rightarrow Y$$

  – E.g. ssn, name → ssn

- **Augmentation**

$$X \rightarrow Y \Rightarrow XW \rightarrow YW$$

  – E.g. ssn → name then ssn grade → name grade

Armstrong's Axioms

- **Transitivity**

$$\left.\begin{aligned} X \rightarrow Y \\ Y \rightarrow Z \end{aligned}\right\} \Rightarrow X \rightarrow Z$$

e.g. if ssn → address and address → tax-rate

then

ssn → tax-rate

Armstrong's Axioms

Reflexivity: $Y \subseteq X \Rightarrow X \rightarrow Y$

Augmentation: $X \rightarrow Y \Rightarrow XW \rightarrow YW$

Transitivity:

$$\left.\begin{array}{l} X \rightarrow Y \\ Y \rightarrow Z \end{array}\right\} \Rightarrow X \rightarrow Z$$

'sound' and 'complete'

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Armstrong Axioms

- Additional rules
  - Union $$\left.\begin{array}{l} X \to Y \\ X \to Z \end{array}\right\} \Rightarrow X \to YZ$$
  - Decomposition $$X \to YZ \Rightarrow \left\{\begin{array}{l} X \to Y \\ X \to Z \end{array}\right.$$
  - Pseudo-transitivity $$\left.\begin{array}{l} X \to Y \\ YW \to Z \end{array}\right\} \Rightarrow XW \to Z$$

Armstrong's Axioms

- Prove 'Union' from three axioms:

$$\left.\begin{matrix} X \rightarrow Y \\ X \rightarrow Z \end{matrix}\right\} \Rightarrow X \rightarrow YZ$$

Armstrong's Axioms

- Prove 'Union' from three axioms:

$$\left.\begin{array}{ll} X \rightarrow Y & (1) \\ X \rightarrow Z & (2) \end{array}\right\}$$

$$(1) + augm.w / Z \Rightarrow XZ \rightarrow YZ \qquad (3)$$

$$(2) + augm.w / X \Rightarrow XX \rightarrow XZ \qquad (4)$$

$$but \quad XX \quad is \quad X \quad thus$$

$$(3) \; + \; (4) \quad and \quad transitivity \quad \Rightarrow X \rightarrow YZ$$

Armstrong's Axioms

- Prove Pseudo-transitivity: try it

$$Y \subseteq X \Rightarrow X \rightarrow Y$$

$$X \rightarrow Y \Rightarrow XW \rightarrow YW$$

$$\left.\begin{array}{l} X \rightarrow Y \\ Y \rightarrow Z \end{array}\right\} \Rightarrow X \rightarrow Z$$

$$\left.\begin{array}{l} X \rightarrow Y \\ YW \rightarrow Z \end{array}\right\} \Rightarrow XW \rightarrow Z$$

Armstrong's Axioms

- Prove Decomposition: try it

$$Y \subseteq X \Rightarrow X \to Y$$

$$X \to Y \Rightarrow XW \to YW$$

$$\left.\begin{array}{l} X \to Y \\ Y \to Z \end{array}\right\} \Rightarrow X \to Z$$

$$X \to YZ \Rightarrow \left.\begin{array}{l} X \to Y \\ X \to Z \end{array}\right\}$$

Note on notation

- Relation Schema: R(A1, A2, A3): parentheses surround attributes, attributes separated by commas.
- Set of attributes: {A1, A2, A3}: curly braces surround attributes, attributes separated by commas
- FD: A1 A2 $\rightarrow$ A3: no parentheses or curly braces, attributes separated by spaces, arrows separates left hand side and right hand side
- Set of FDs: {A1 A2 $\rightarrow$ A3, A2 $\rightarrow$ A1}: curly braces surround FDs, FDs separated by commas

Overview

- Functional dependencies
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  - Armstrong' s "axioms"
  - **FD closure and cover**

FDs - Closure F+

Given a set F of FD (on a schema)

F+ is the set of all implied FD. Eg.,

takes(ssn, c-id, grade, name, address)

$$\left.\begin{array}{l}\text{ssn, c-id -> grade} \\ \text{ssn-> name, address}\end{array}\right\} \mathbf{F}$$

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FDs - Closure F+

ssn, c-id -> grade

ssn-> name, address

ssn-> ssn

ssn, c-id-> address

c-id, address-> c-id

...

F+

Computing Closures of FDs

- To compute the closure of a set of FDs, repeatedly apply Armstrong's Axioms until you cannot find any new FDs

Examples of Computing Closures of FDs

- ((Let us include only completely non-trivial FDs in these examples, with a single attribute on the right))
- F = {A→B, B→C}
- {F}+ = ??

Examples of Computing Closures of FDs

- ((Let us include only completely non-trivial FDs in these examples, with a single attribute on the right))
- F = {A→B, B→C}
- {F}+ = {A→B, B→C, A→C, AC→B, AB→C}

Examples of Computing Closures of FDs

- ((Let us include only completely non-trivial FDs in these examples, with a single attribute on the right))
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- F = {AB→C, BC→A, AC→B}
- {F}+ = {AB→C, BC→A, AC→B}

Examples of Computing Closures of FDs

- ((Let us include only completely non-trivial FDs in these examples, with a single attribute on the right))
- F = {A→B, B→C, C→D}
- {F}+ = ??

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Examples of Computing Closures of FDs

- ((Let us include only completely non-trivial FDs in these examples, with a single attribute on the right))
- F = {A→B, B→C, C→D}
- {F}+ = {A→B, B→C, C→D, A→C, A→D, B→D, …}

Closures of Attributes vs Closure of FDs

- Both algorithms take as input a relation R and a set of FDs F
- Closure of FDs:
  - Computes {F}+, the **set of all FDs** that follow from F
  - Output is a set of FDs
  - Output may contain an exponential number of FDs
- Closure of attributes:
  - In addition, takes a set {A1, A2…, An} of attributes as input
  - Computes {A1, A2, …, An}+, the **set of all attributes** B, such that A1 A2 … An → B follows from F
  - Output is set of all attributes
  - Output may contain at most the number of attributes in R

FDs - 'canonical cover' Fc

Given a set F of FD (on a schema)

Fc is a **minimal set** of equivalent FDs. Eg.,

takes(ssn, c-id, grade, name, address)

ssn, c-id -> grade

ssn-> name, address

ssn,name-> name, address

ssn, c-id-> grade, name

F

Canonical cover

- Also sometimes called the ‘minimal basis’ or ‘minimal cover’

FDs - ‘canonical cover’ Fc

Fc

$$\boxed{\begin{array}{l}\text{ssn, c-id -> grade} \\ \text{ssn-> name, address}\end{array}}$$

ssn,name-> name, address

ssn, c-id-> grade, name

F

FDs - 'canonical cover' Fc

- why do we need it?
- define it properly
- compute it efficiently

FDs - ‘canonical cover’ Fc

- why do we need it?
  - easier to compute candidate keys
- define it properly
- compute it efficiently

FDs - 'canonical cover' Fc

- define it properly - three properties
  - 1) the RHS of every FD is a single attribute
  - 2) the closure of Fc is identical to the closure of F (ie., Fc and F are equivalent)
  - 3) Fc is minimal (ie., if we eliminate any attribute from the LHS or RHS of a FD, property #2 is violated

FDs - ‘canonical cover’ Fc

- #3: we need to eliminate ‘extraneous’ attributes. An attribute is ‘extraneous if
  - the closure is the same, before and after its elimination
  - or if F-before implies F-after and vice-versa

FDs - 'canonical cover' Fc

ssn, c-id -> grade

ssn-> name, address

ssn,~~name~~-> name, address

ssn, c-id-> grade, ~~name~~

F

FDs - 'canonical cover' Fc

Algorithm:

- examine each FD; drop extraneous LHS or RHS attributes; or redundant FDs
- make sure that FDs have a single attribute in their RHS
- repeat until no change

FDs - 'canonical cover' Fc

Trace algo for

AB->C (1)

A->BC (2)

B->C (3)

A->B (4)

FDs - 'canonical cover' Fc

Trace algo for

AB->C (1)

A->BC (2)

B->C (3)

A->B (4)

split (2):

AB->C (1)
A->B (2')
A->C (2'')
B->C (3)
A->B (4)

FDs - 'canonical cover' Fc

AB->C (1)
~~A->B (2')~~
A->C (2'')
B->C (3)
A->B (4)

AB->C (1)

A->C (2'')
B->C (3)
A->B (4)

FDs - 'canonical cover' Fc

AB->C (1)

A->C (2'')
B->C (3)
A->B (4)

(2''): redundant (implied by (4), (3) and transitivity

AB->C (1)

B->C (3)
A->B (4)

FDs - ‘canonical cover’ Fc

AB->C (1)	B->C (1’)
B->C (3)	B->C (3)
A->B (4)	A->B (4)

in (1), ‘A’ is extraneous:
(1),(3),(4) imply
(1’),(3),(4), and vice versa

FDs - ‘canonical cover’ Fc

~~B->C (1')~~

B->C (3)
A->B (4)

B->C (3)
A->B (4)

- nothing is extraneous
- all RHS are single attributes
- final and original set of FDs are equivalent (same closure)

FDs - ‘canonical cover’ Fc

BEFORE

AB->C (1)
A->BC (2)
B->C (3)
A->B (4)

AFTER

B->C (3)
A->B (4)

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