

# CS 4604: Introduction to Database Management Systems

*B. Aditya Prakash*

Lecture #11: Query Optimization

# Notes

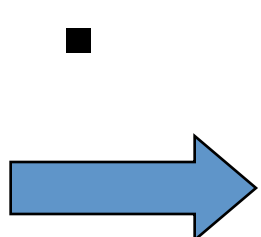
- Some parts from (a copy of the paper is on the course webpage)
  - Selinger, Patricia, M. Astrahan, D. Chamberlin, Raymond Lorie, and T. Price. "Access Path Selection in a Relational Database Management System." In Proceedings of ACM SIGMOD, Boston, MA, 1979, pp. 22-34.



# Cost-based Query Sub-System

Queries

```
Select *  
From Blah B  
Where B.blah = blah
```



Query Parser

Query Optimizer

Plan Generator ↔ Plan Cost Estimator

Query Plan Evaluator

Usually there is a heuristics-based rewriting step before the cost-based steps.

Catalog Manager



# Multiple Algorithms: Range Searches

- Sequential Scan
  - Hashes
  - B-Trees
  - ....
- 
- Saw some of them in previous lectures

# Multiple Algorithms: Joins

- Merge-Join (like merge-sort)
- Hash-Join (using hashes)
- Indexed-Join (using indexes)
- Nested loops Join (most obvious)
- ....
  
- Saw some of them in previous lectures

# Why Query optimization?

- SQL: ~declarative
- good q-opt -> big difference
  - eg., seq. Scan vs
  - B-tree index, on  $P=1,000$  pages

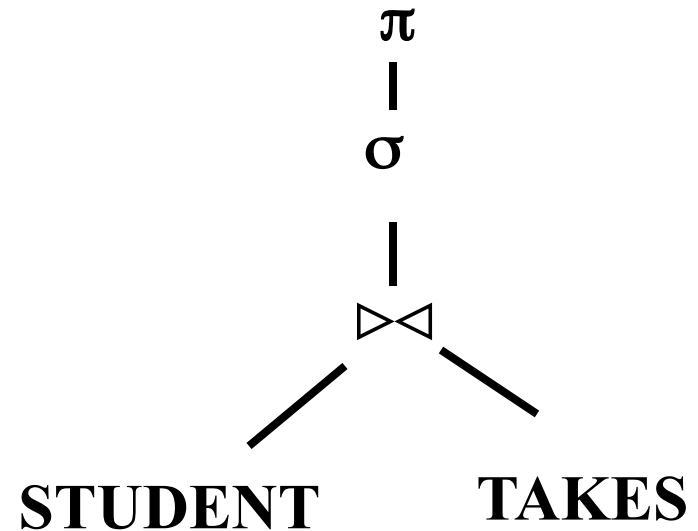
# Q-opt steps

- bring query in internal form (eg., parse tree)
- ... into ‘canonical form’ (syntactic q-opt)
- generate alt. plans
- estimate cost; pick best

# Q-opt - example

```

select name
from STUDENT, TAKES
where c-id = '4604' and
STUDENT.ssn = TAKES.ssn
  
```

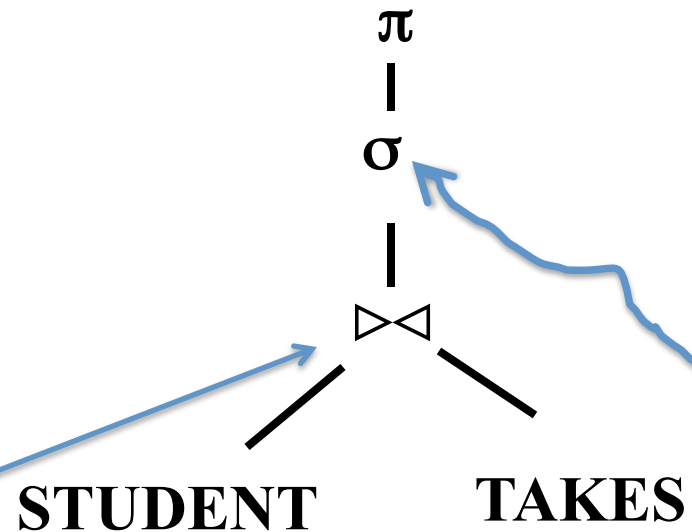




# Q-opt - example

```

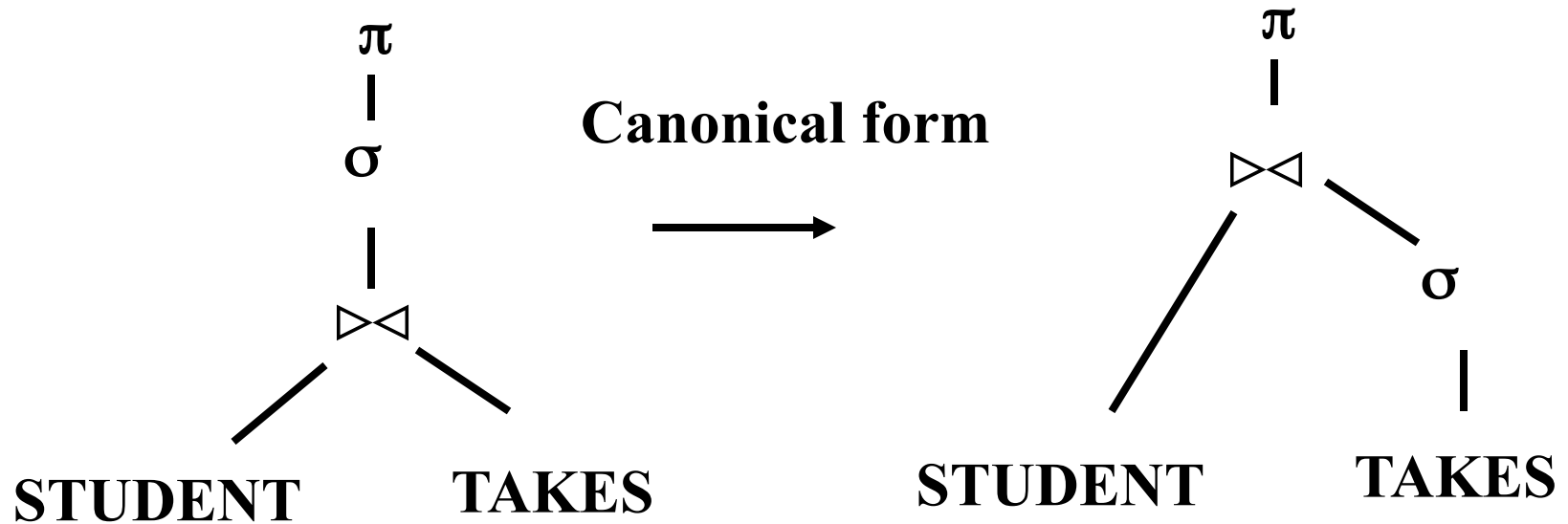
select name
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```



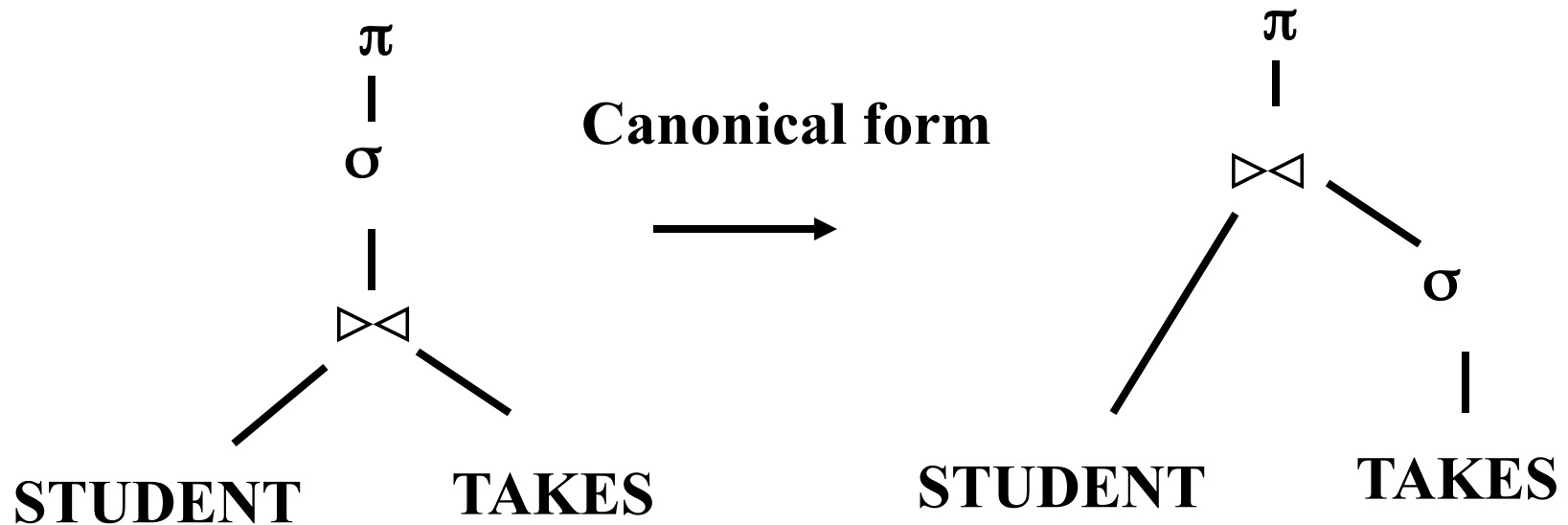
**Join Predicate => STUDENT.ssn = TAKES.ssn  
(is assumed to be part of the join)**

**Non-join Predicate => c-id = '4604'  
(part of the explicit selection)**

# Q-opt - example



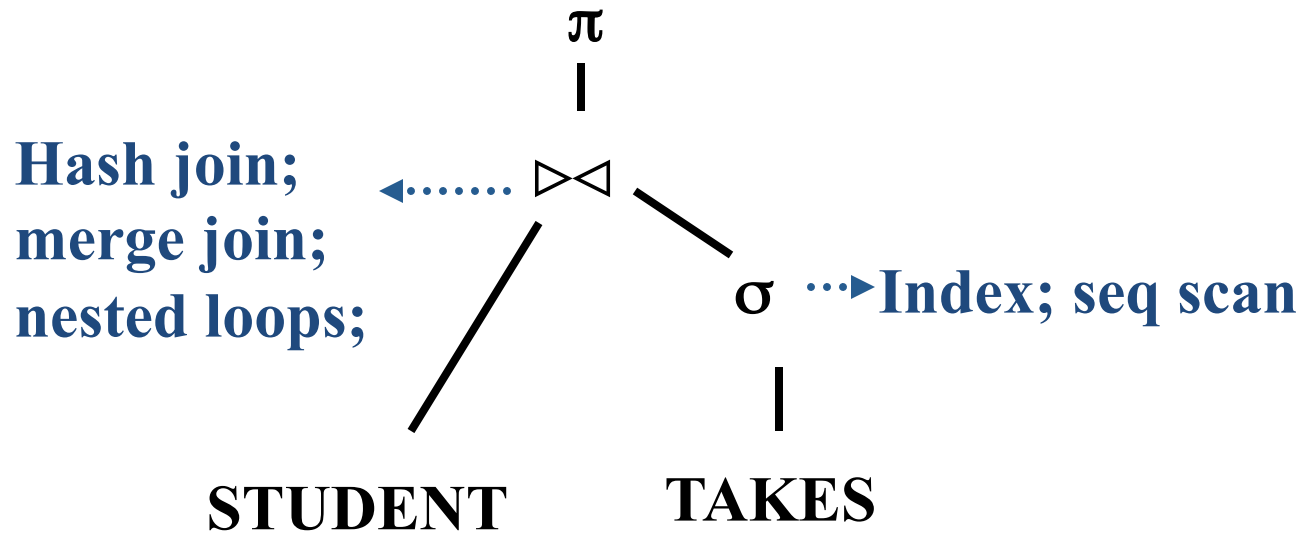
# Q-opt - example



**Canonical Form has the following properties:**

- 1. Push Selections as much as possible.**
- 2. Push Projections as much as possible**
- 3. It is a left-deep join tree (we will see this later)**

# Q-opt - example



# Equivalence of expressions

- A.k.a.: syntactic q-opt
- in short: perform selections and projections early

# Equivalence of expressions

- Q: How to prove a transformation rule?

$$\sigma_P(R1 \bowtie R2) \stackrel{?}{=} \sigma_P(R1) \bowtie \sigma_P(R2)$$

- A: use RA, to show that LHS = RHS, eg:

$$\sigma_P(R1 \cup R2) \stackrel{?}{=} \sigma_P(R1) \cup \sigma_P(R2)$$

# Equivalence of expressions

$$\sigma_P(R1 \cup R2) \stackrel{?}{=} \sigma_P(R1) \cup \sigma_P(R2)$$

$$t \in LHS \Leftrightarrow$$

$$t \in (R1 \cup R2) \wedge P(t) \Leftrightarrow$$

$$(t \in R1 \vee t \in R2) \wedge P(t) \Leftrightarrow$$

$$(t \in R1 \wedge P(t)) \vee (t \in R2) \wedge P(t)) \Leftrightarrow$$

# Equivalence of expressions

$$\sigma_P(R1 \cup R2) \stackrel{?}{=} \sigma_P(R1) \cup \sigma_P(R2)$$

...

$$(t \in R1 \wedge P(t)) \quad \vee \quad (t \in R2) \wedge P(t) \quad \Leftrightarrow$$

$$(t \in \sigma_P(R1)) \quad \vee \quad (t \in \sigma_P(R2)) \quad \Leftrightarrow$$

$$t \in \sigma_P(R1) \cup \sigma_P(R2) \quad \Leftrightarrow$$

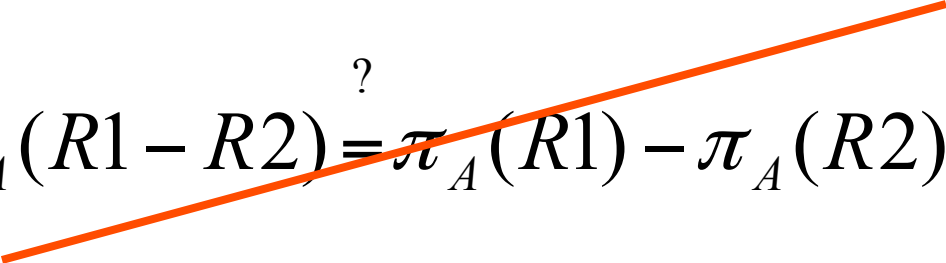
$t \in RHS$


*QED*



# Equivalence of expressions

- Q: how to disprove a rule??

$$\pi_A(R1 - R2) \stackrel{?}{=} \pi_A(R1) - \pi_A(R2)$$




**Construct a  
counter-example!**

# Equivalence of expressions

## ■ Selections

– perform them early

– break a complex predicate, and push

$$\sigma_{p1 \wedge p2 \wedge \dots \wedge pn}(R) = \sigma_{p1}(\sigma_{p2}(\dots \sigma_{pn}(R)\dots))$$

– simplify a complex predicate

- ( 'X=Y and Y=3' ) -> 'X=3 and Y=3'

# Equivalence of expressions

- Projections
  - perform them early (but carefully...)
    - Smaller tuples
    - Fewer tuples (if duplicates are eliminated)
  - project out all attributes except the ones requested or required (e.g., joining attr.)

# Equivalence of expressions

- Joins

- Commutative , associative

$$R \bowtie S = S \bowtie R$$

$$(R \bowtie S) \bowtie T = R \bowtie (S \bowtie T)$$

- Q: n-way join - how many diff. orderings?

# Equivalence of expressions

- Joins - Q: n-way join - how many diff. orderings?
- A: Catalan number  $\sim 4^n$ 
  - Exhaustive enumeration: too slow.

# (Some) Transformation Rules (1)

1. Conjunctive selection operations can be deconstructed into a sequence of individual selections.

$$\sigma_{\theta_1 \wedge \theta_2}(E) = \sigma_{\theta_1}(\sigma_{\theta_2}(E))$$

2. Selection operations are commutative.

$$\sigma_{\theta_1}(\sigma_{\theta_2}(E)) = \sigma_{\theta_2}(\sigma_{\theta_1}(E))$$

3. Only the last in a sequence of projection operations is needed, the others can be omitted.

$$\Pi_{L_1}(\Pi_{L_2}(\dots(\Pi_{L_n}(E))\dots)) = \Pi_{L_1}(E)$$

4. Selections can be combined with Cartesian products and theta joins.

- a.  $\sigma_{\theta}(E_1 \times E_2) = E_1 \bowtie_{\theta} E_2$

- b.  $\sigma_{\theta_1}(E_1 \bowtie_{\theta_2} E_2) = E_1 \bowtie_{\theta_1 \wedge \theta_2} E_2$

# (Some) Transformation Rules (2)

5. Theta-join operations (and natural joins) are commutative.

$$E_1 \bowtie_{\theta} E_2 = E_2 \bowtie_{\theta} E_1$$

6. (a) Natural join operations are associative:

$$(E_1 \bowtie E_2) \bowtie E_3 = E_1 \bowtie (E_2 \bowtie E_3)$$

(b) Theta joins are associative in the following manner:

$$(E_1 \bowtie_{\theta_1} E_2) \bowtie_{\theta_2 \wedge \theta_3} E_3 = E_1 \bowtie_{\theta_1 \wedge \theta_3} (E_2 \bowtie_{\theta_2} E_3)$$

where  $\theta_2$  involves attributes from only  $E_2$  and  $E_3$ .

# (Some) Transformation Rules (3)

7. The selection operation distributes over the theta join operation under the following two conditions:
- (a) When all the attributes in  $\theta_0$  involve only the attributes of one of the expressions ( $E_1$ ) being joined.

$$\sigma_{\theta_0}(E_1 \bowtie_{\theta} E_2) = (\sigma_{\theta_0}(E_1)) \bowtie_{\theta} E_2$$

- (b) When  $\theta_1$  involves only the attributes of  $E_1$  and  $\theta_2$  involves only the attributes of  $E_2$ .

$$\sigma_{\theta_1 \wedge \theta_2}(E_1 \bowtie_{\theta} E_2) = (\sigma_{\theta_1}(E_1)) \bowtie_{\theta} (\sigma_{\theta_2}(E_2))$$



## Q-opt steps

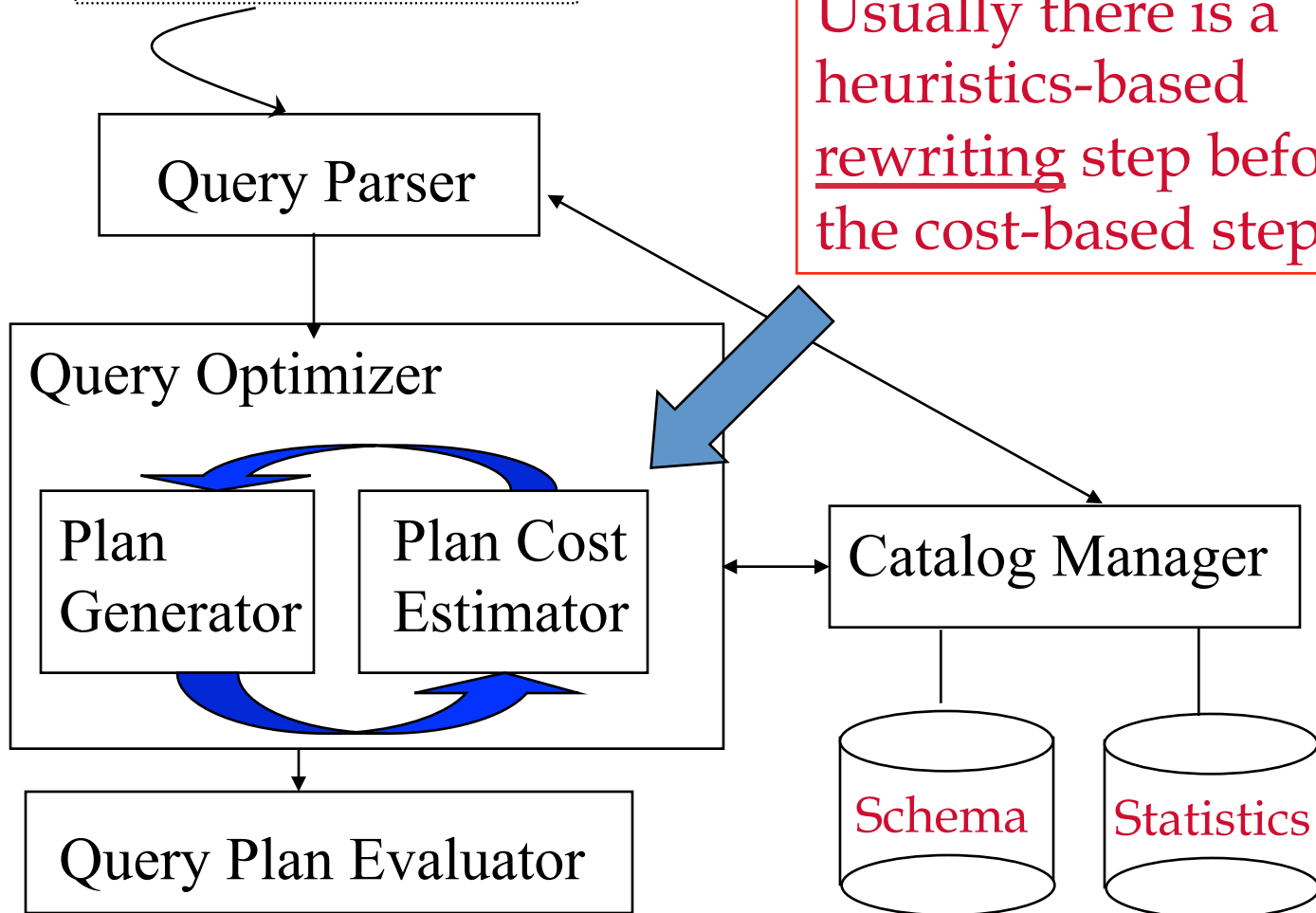
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# Cost-based Query Sub-System

## Queries

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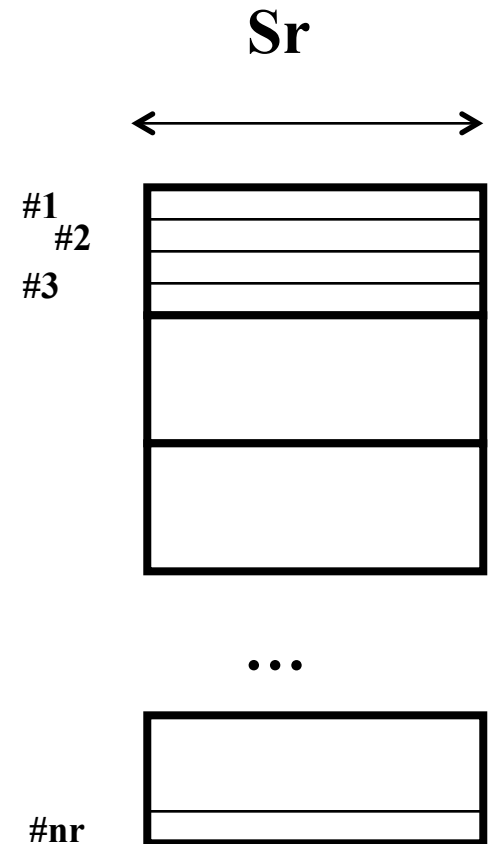


# Cost estimation

- Eg., find ssn' s of students with an 'A' in 4604 (using seq. scanning)
- How long will a query take?
  - CPU (but: small cost; decreasing; tough to estimate)
  - Disk (mainly, # block transfers)
- How many tuples will qualify?
- (what statistics do we need to keep?)

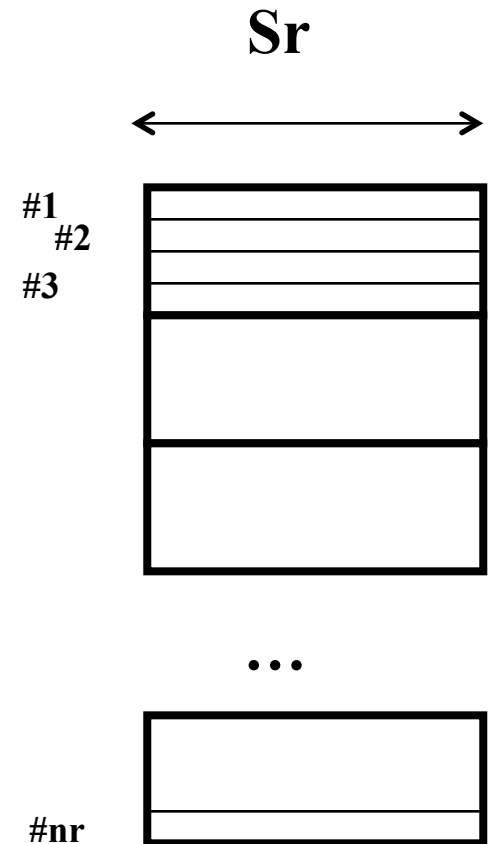
# Cost estimation

- Statistics: for each relation 'r' we keep
  - nr : # tuples;
  - Sr : size of tuple in bytes



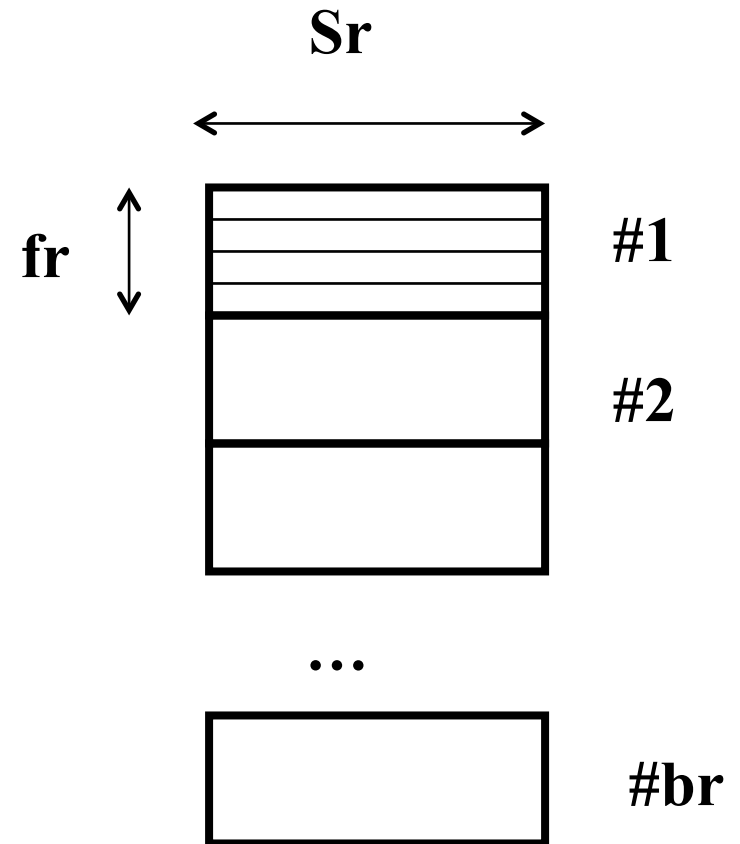
# Cost estimation

- Statistics: for each relation 'r' we keep
  - ...
  - $V(A,r)$ : number of distinct values of attr. 'A'
  - (recently, histograms, too)



# Derivable statistics

- blocking factor = max# records/block (=?? )
- br: # blocks (=?? )
- $SC(A,r)$  = selection cardinality = avg# of records with A=given (=?? )



# Derivable statistics

- blocking factor = max# records/block (=  $B/S_r$  ;  
B: block size in bytes)
- br: # blocks (=  $nr / (\text{blocking-factor})$  )

# Derivable statistics

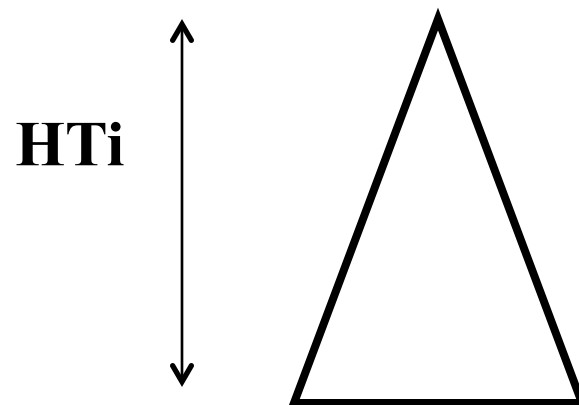
- $SC(A,r)$  = selection cardinality = avg# of records with  $A$ =given ( $= nr / V(A,r)$ ) (assumes uniformity...)

eg: 10,000 students, 10 departments – how many students in CS?



# Additional quantities we need:

- For index ‘i’ :
  - $f_i$ : average fanout ( $\sim 50-100$ )
  - $HT_i$ : # levels of index ‘i’ ( $\sim 2-3$ )
    - $\sim \log(\#entries)/\log(f_i)$
  - $LBI$ : # blocks at leaf level



# Statistics

- Where do we store them?
- How often do we update them?

## Q-opt steps

- bring query in internal form (eg., parse tree)
- ... into ‘canonical form’ (syntactic q-opt)
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# Selections

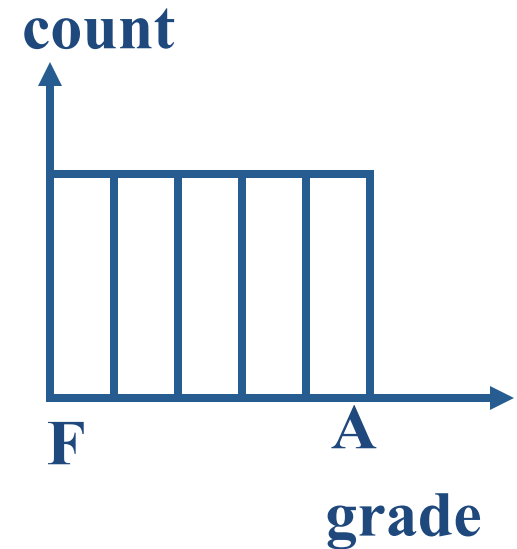
- we saw simple predicates ( $A = \text{constant}$ ; eg., ‘name=Smith’ )
- how about more complex predicates, like
  - ‘salary > 10K’
  - ‘age = 30 and job-code=“analyst” ’
- what is their selectivity?

# Selections – complex predicates

- selectivity  $\text{sel}(P)$  of predicate  $P$  :
  - == fraction of tuples that qualify
  - $\text{sel}(P) = \text{SC}(P) / nr$

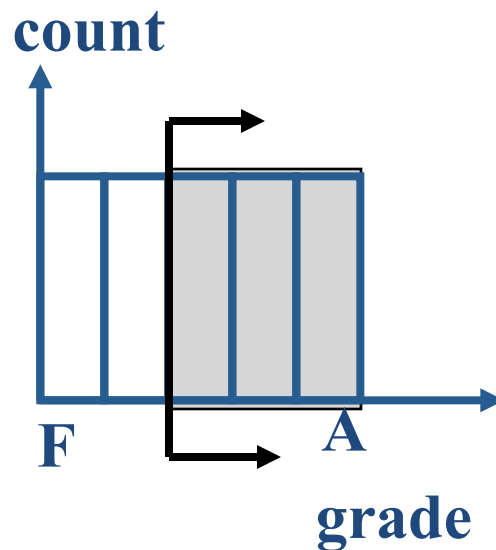
# Selections – complex predicates

- eg., assume that  $V(\text{grade, TAKES})=5$  distinct values
- simple predicate  $P: A=\text{constant}$ 
  - $\text{sel}(A=\text{constant}) = 1/V(A,r)$
  - eg.,  $\text{sel}(\text{grade} = \text{'B'}) = 1/5$
- (what if  $V(A,r)$  is unknown??)



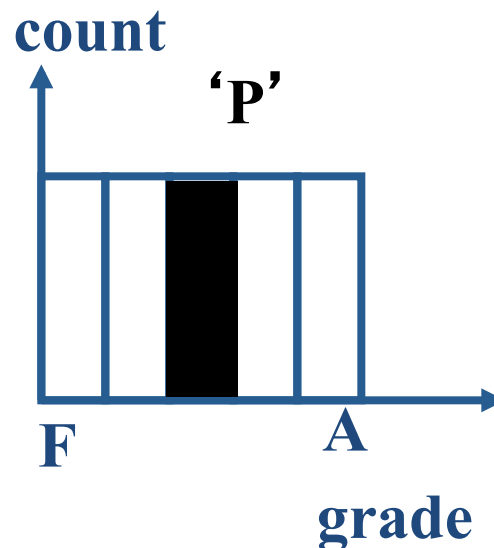
# Selections – complex predicates

- range query:  $\text{sel}(\text{grade} \geq \text{'C'})$ 
  - $\text{sel}(A > a) = (\text{Amax} - a) / (\text{Amax} - \text{Amin})$



# Selections - complex predicates

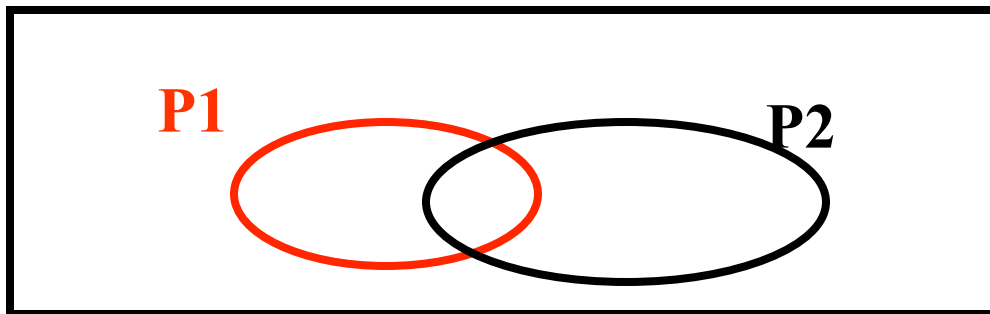
- negation:  $\text{sel}(\text{grade} \neq \text{'C'})$ 
  - $\text{sel}(\text{not } P) = 1 - \text{sel}(P)$
  - (Observation: selectivity  $\approx$  probability)





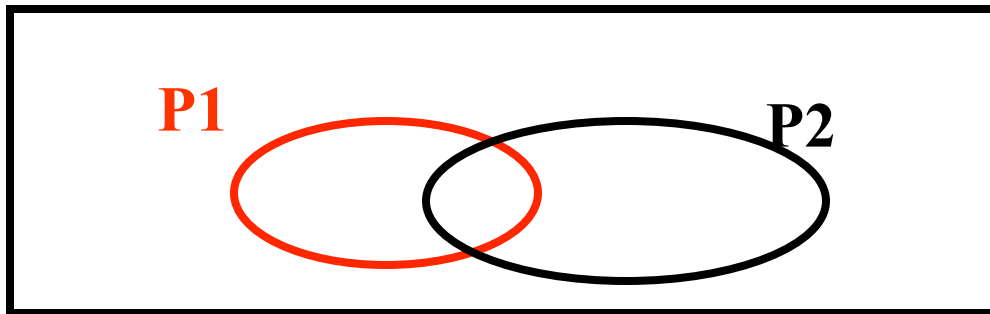
# Selections - complex predicates

- Conjunction:
  - $\text{sel}(\text{grade} = \text{'C'} \text{ and } \text{course} = \text{'4604'})$
  - $\text{sel}(P1 \text{ and } P2) = \text{sel}(P1) * \text{sel}(P2)$
  - INDEPENDENCE ASSUMPTION



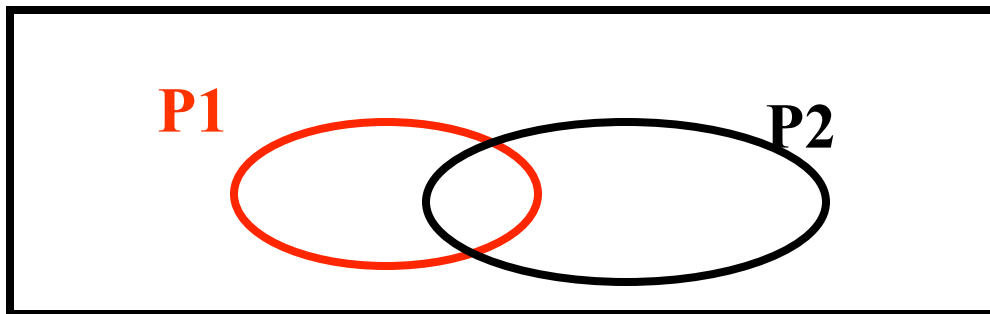
# Selections - complex predicates

- Disjunction:
  - $\text{sel}(\text{grade} = \text{'C'} \text{ or } \text{course} = \text{'4604'})$
  - $\text{sel}(P1 \text{ or } P2) = \text{sel}(P1) + \text{sel}(P2) - \text{sel}(P1 \text{ and } P2)$
  - $= \text{sel}(P1) + \text{sel}(P2) - \text{sel}(P1) * \text{sel}(P2)$
  - INDEPENDENCE ASSUMPTION, again



# Selections - complex predicates

- disjunction: in general
  - $\text{sel}(P1 \text{ or } P2 \text{ or } \dots Pn) =$   
 $1 - (1 - \text{sel}(P1)) * (1 - \text{sel}(P2)) * \dots (1 - \text{sel}(Pn))$



# Selections Selectivity – summary

- $\text{sel}(A=\text{constant}) = 1/V(A,r)$
- $\text{sel}(A > a) = (A_{\max} - a) / (A_{\max} - A_{\min})$
- $\text{sel}(\text{not } P) = 1 - \text{sel}(P)$
- $\text{sel}(P1 \text{ and } P2) = \text{sel}(P1) * \text{sel}(P2)$
- $\text{sel}(P1 \text{ or } P2) = \text{sel}(P1) + \text{sel}(P2) - \text{sel}(P1)*\text{sel}(P2)$
- $\text{sel}(P1 \text{ or } \dots \text{ or } Pn) = 1 - (1-\text{sel}(P1))*\dots*(1-\text{sel}(Pn))$
  
- UNIFORMITY and INDEPENDENCE ASSUMPTIONS

# Result Size Estimation for Joins

- Q: Given a join of R and S, what is the range of possible result sizes (in #of tuples)?
  - Hint: what if  $R\_cols \cap S\_cols = \emptyset$ ?
  - $R\_cols \cap S\_cols$  is a key for R (and a Foreign Key in S)?

# Result Size Estimation for Joins

- General case:  $R\_cols \cap S\_cols = \{A\}$  (and A is key for neither)

- match each R-tuple with S-tuples

$$\begin{aligned} \text{est\_size} &\lesssim \text{NTuples}(R) * \text{NTuples}(S) / \text{NKeys}(A, S) \\ &\lesssim nr * ns / V(A, S) \end{aligned}$$

- symmetrically, for S:

$$\begin{aligned} \text{est\_size} &\lesssim \text{NTuples}(R) * \text{NTuples}(S) / \text{NKeys}(A, R) \\ &\lesssim nr * ns / V(A, R) \end{aligned}$$

- Overall:

$$\text{est\_size} = \text{NTuples}(R) * \text{NTuples}(S) / \text{MAX}\{\text{NKeys}(A, S), \text{NKeys}(A, R)\}$$

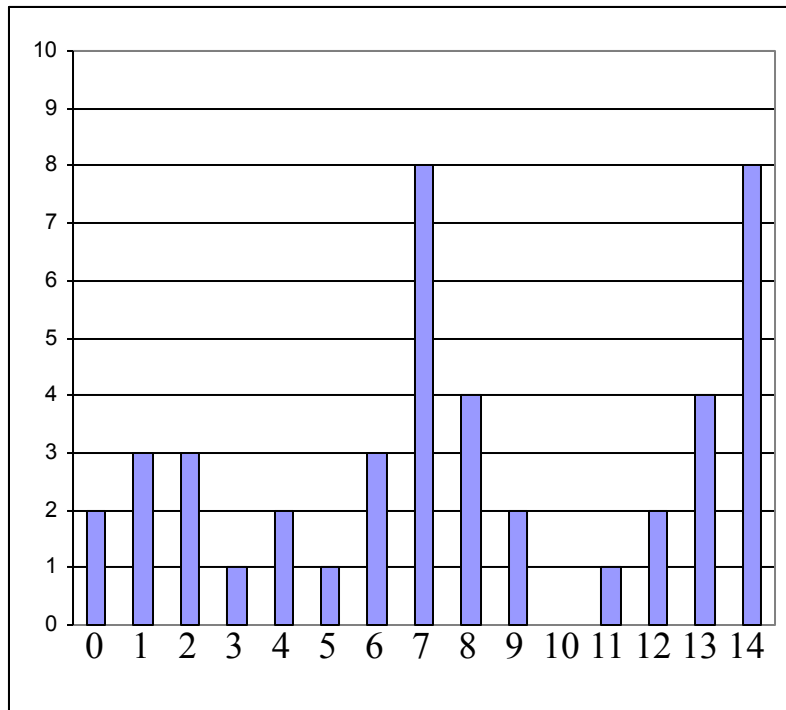




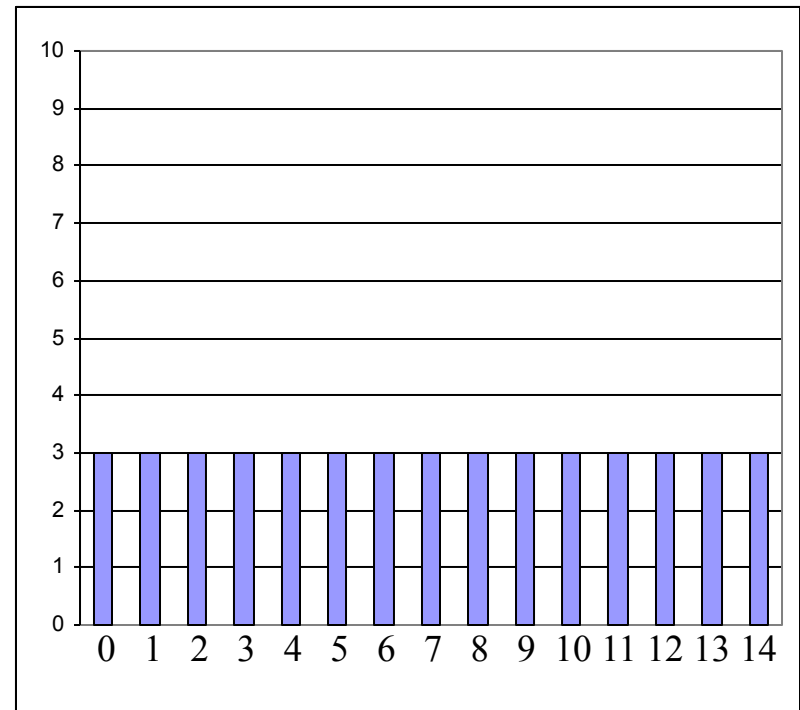
# On the Uniform Distribution Assumption

- Assuming uniform distribution is rather crude

Distribution D



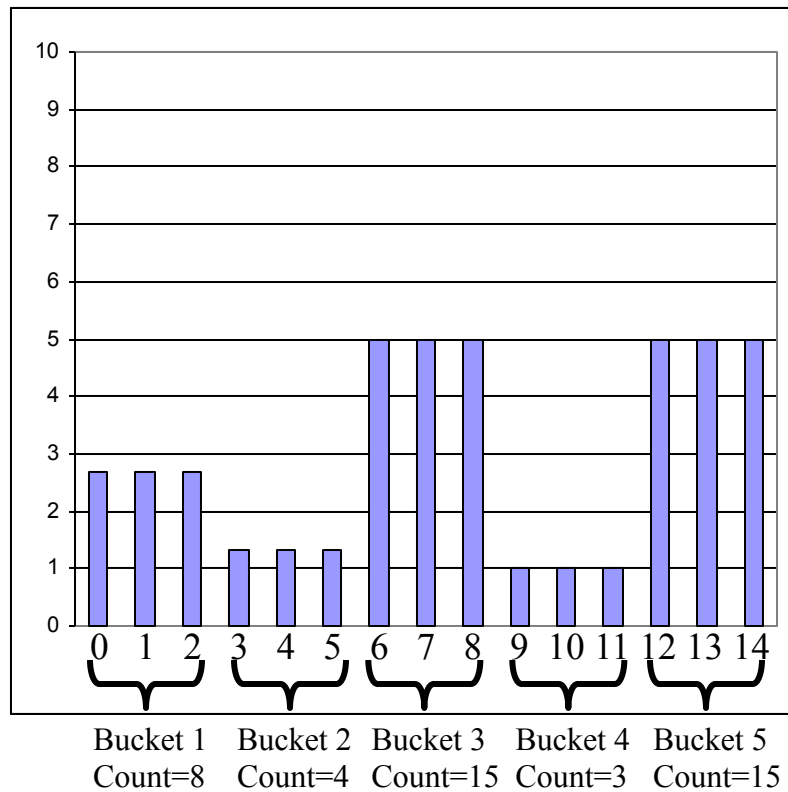
Uniform distribution approximating D



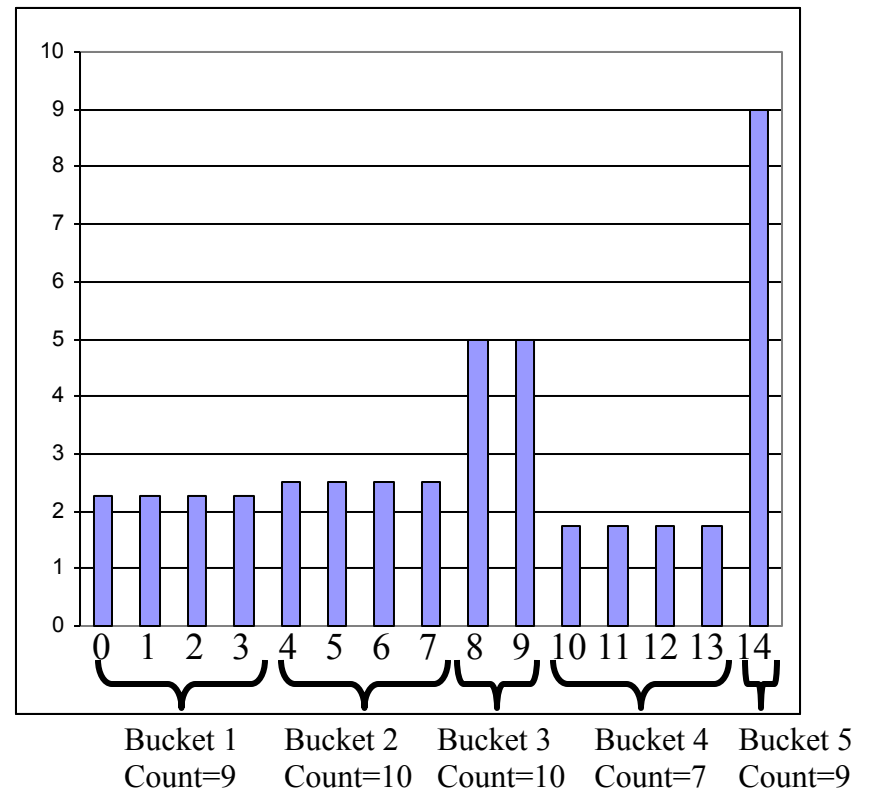
# Histograms

- For better estimation, use a *histogram*

## Equiwidth histogram



## Equidepth histogram ~ quantiles



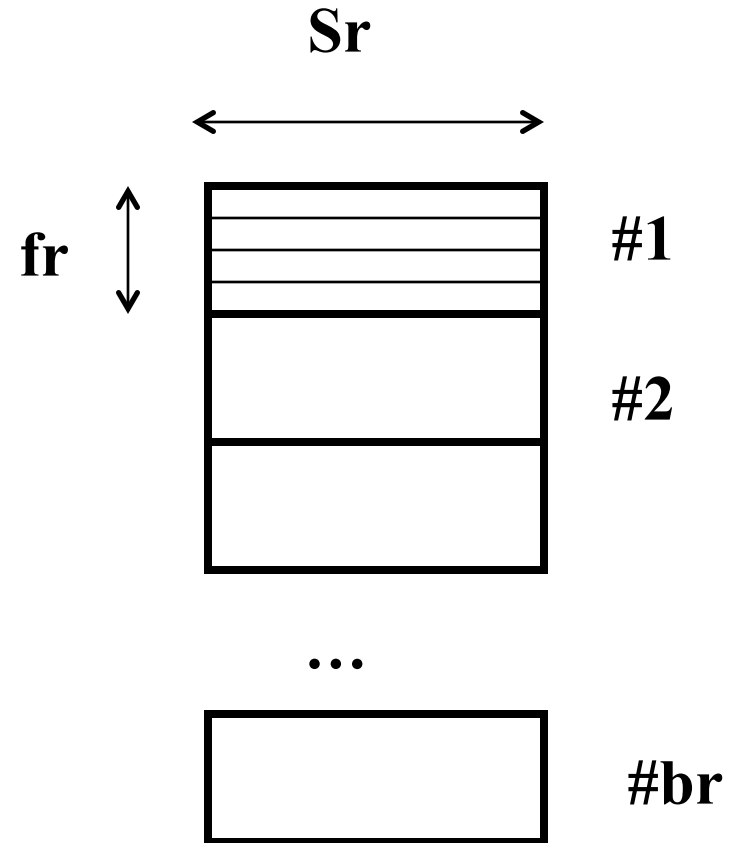


# Q-opt Steps

- bring query in internal form (eg., parse tree)
- ... into 'canonical form' (syntactic q-opt)
- **generate alt. plans**
  - single relation
  - multiple relations
- estimate cost; pick best

# plan generation

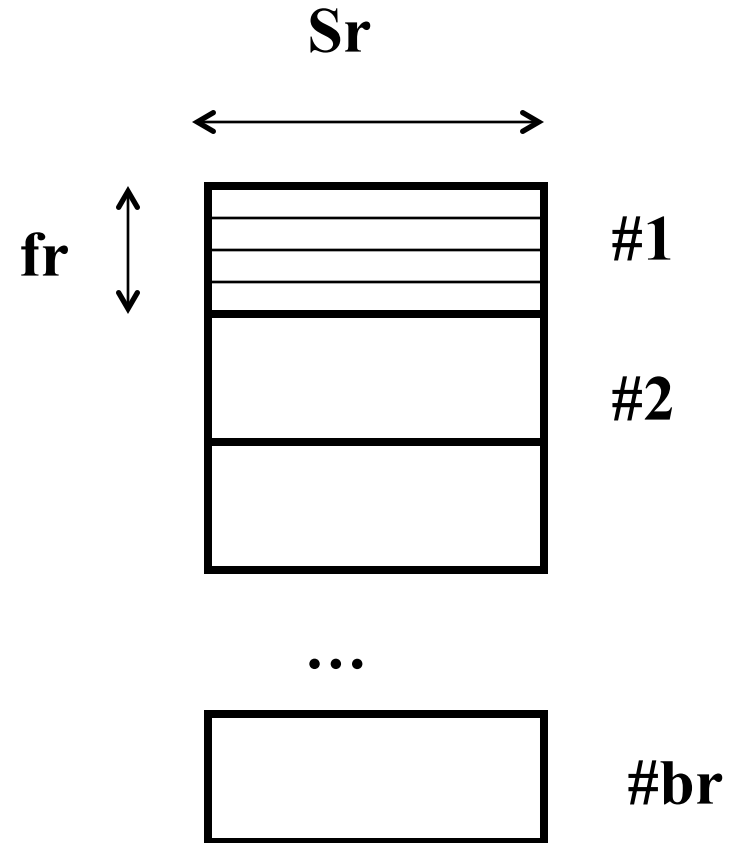
- Selections – eg.,  
**select \***  
**from TAKES**  
**where grade = 'A'**
- Plans?



# plan generation

## ■ Plans?

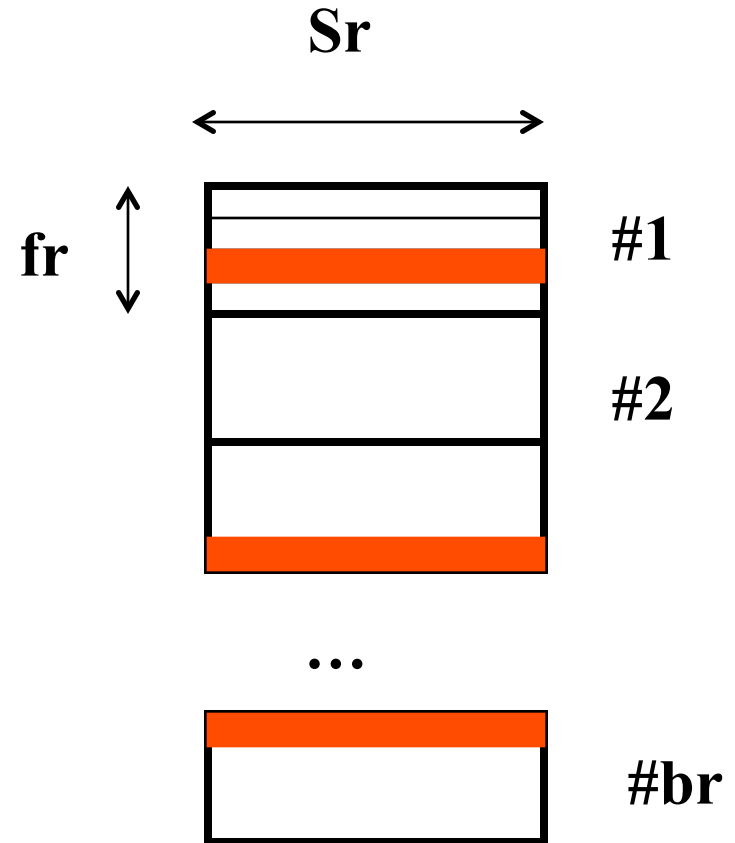
- seq. scan
- binary search
  - (if sorted & consecutive)
- index search
  - if an index exists



# plan generation

seq. scan – cost?

- $br$  (worst case)
- $br/2$  (average, if we search for primary key)

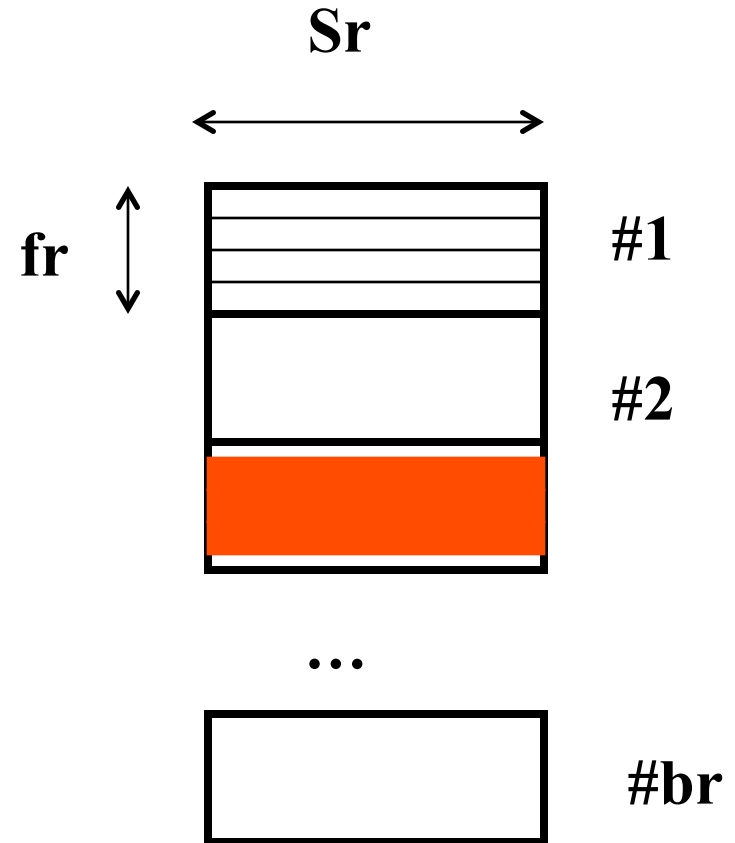


# plan generation

binary search – cost?

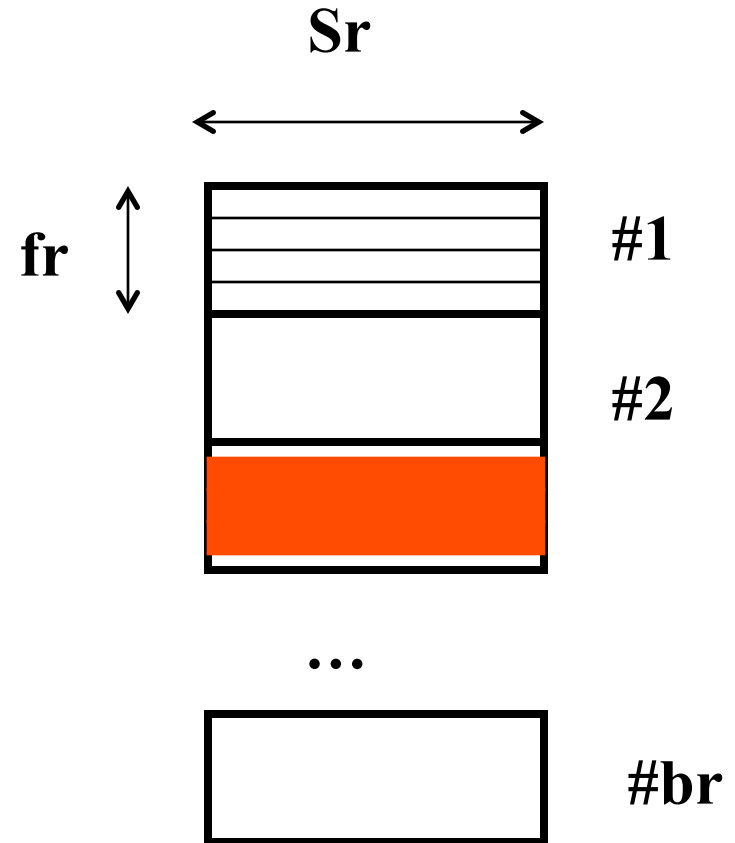
if sorted and  
consecutive:

- $\sim \log(br) +$
- $SC(A,r)/fr$  (=blocks spanned by qual. tuples)



# plan generation

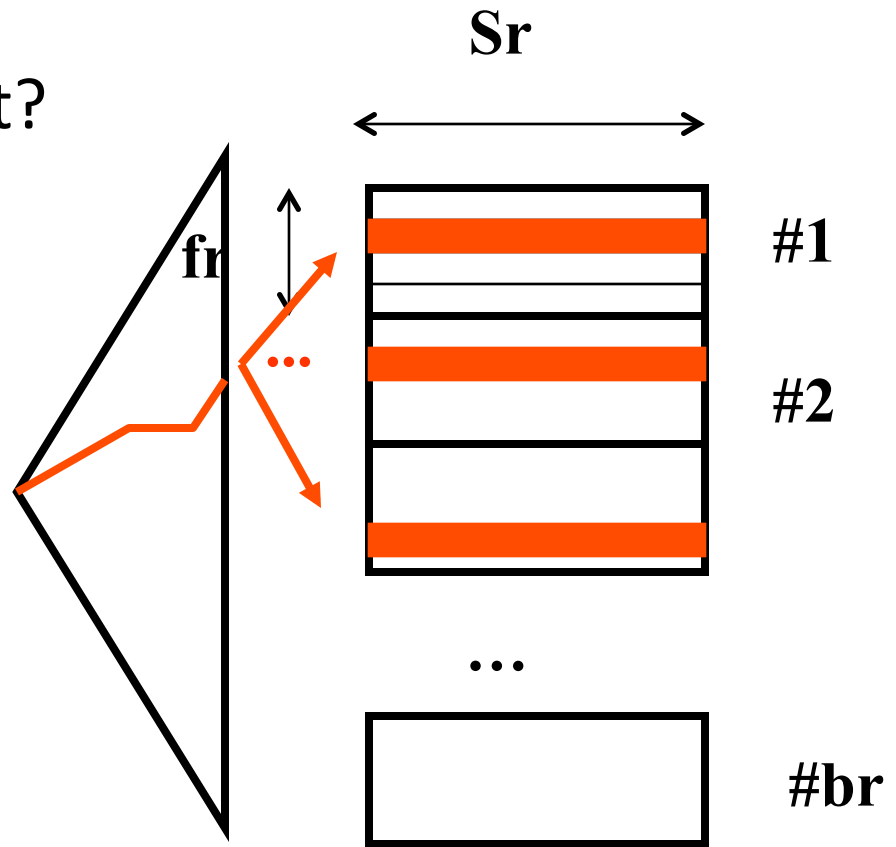
estimation of selection  
 cardinalities  $SC(A,r)$ :  
 – we saw it earlier how  
 to do it for general  
 conditions



# plan generation

method#3: index – cost?

- Roughly  $\log(N)$ , but exact cost tricky



# Q-opt Steps

- bring query in internal form (eg., parse tree)
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  - single relation
  - **multiple relations**
- estimate cost; pick best

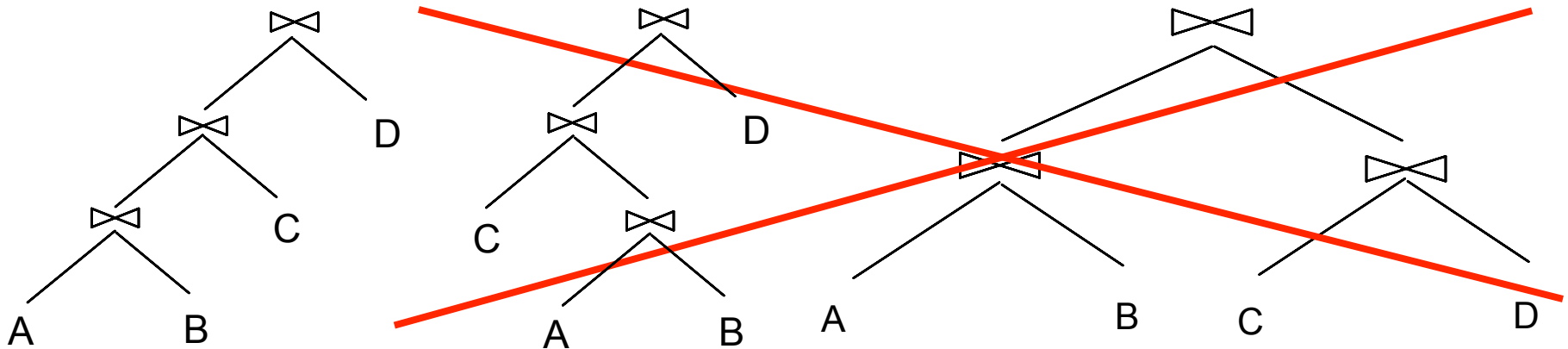


# n-way joins

- $r_1 \text{ JOIN } r_2 \text{ JOIN } \dots \text{ JOIN } r_n$
- typically, break problem into 2-way joins
  - choose between NL, sort merge, hash join, ...

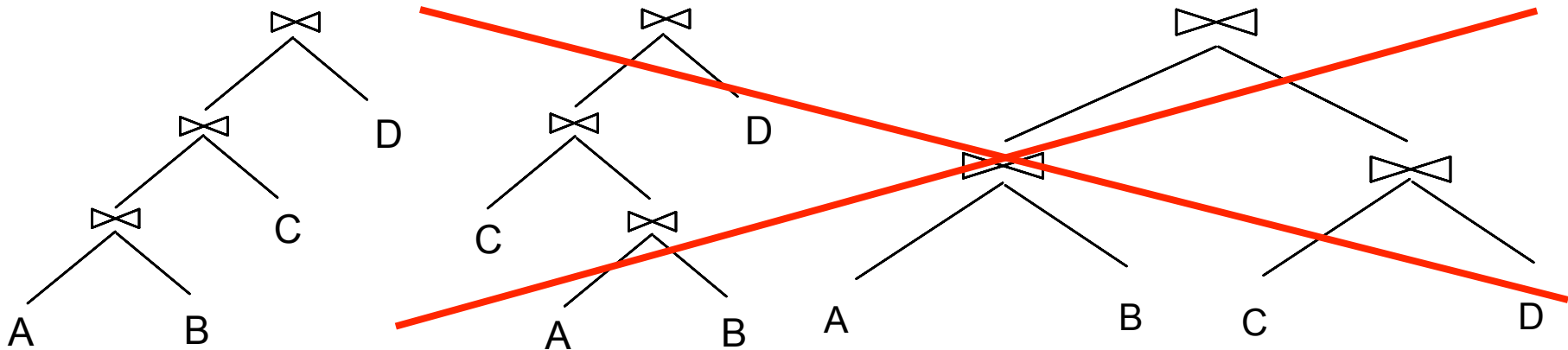
# Queries Over Multiple Relations

- As number of joins increases, number of alternative plans grows rapidly → *need to restrict search space*
- Fundamental decision in System R (IBM): only left-deep join trees are considered. Advantages?



# Queries Over Multiple Relations

- As number of joins increases, number of alternative plans grows rapidly → *need to restrict search space*
- Fundamental decision in System R (IBM): only left-deep join trees are considered. Advantages?
  - fully pipelined* plans.
    - Intermediate results not written to temporary files.



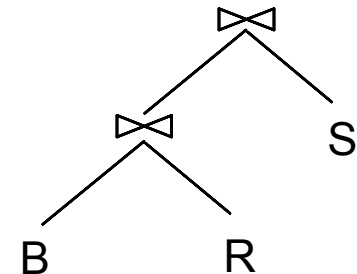
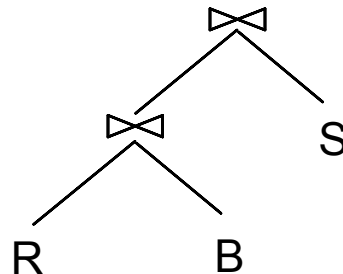
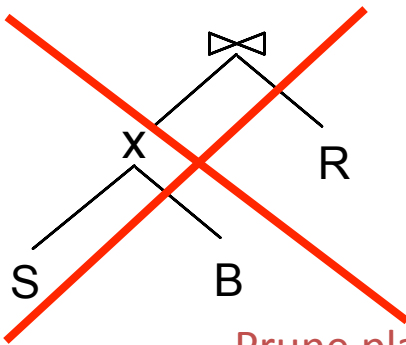
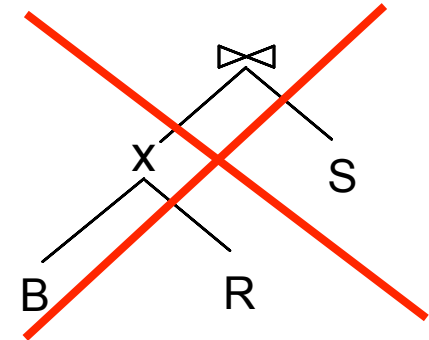
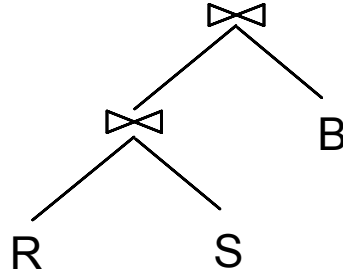
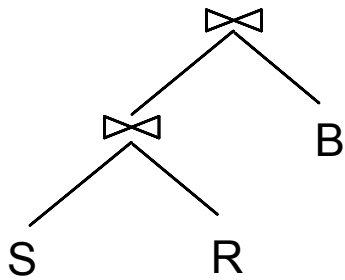
# Queries over Multiple Relations

- Enumerate the orderings (= left deep tree)
- enumerate the plans for each operator
- enumerate the access paths for each table

Dynamic programming, to save cost estimations  
(we wont cover exact algorithm in class)

```
SELECT S.sname, B.bname, R.day
FROM Sailors S, Reserves R, Boats B
WHERE S.sid = R.sid AND R.bid = B.bid
```

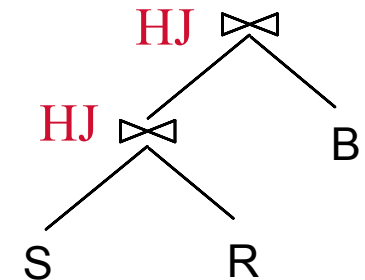
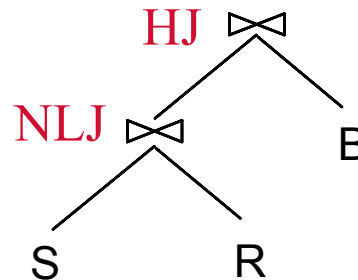
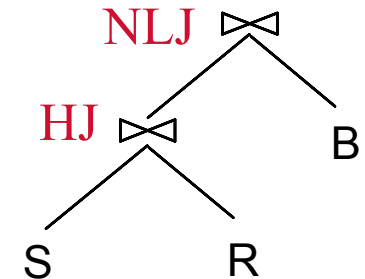
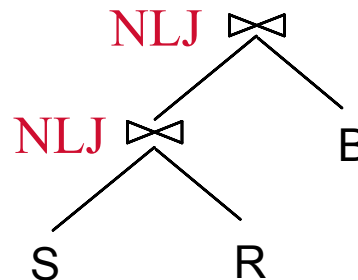
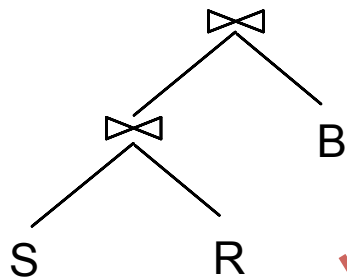
# 1. Enumerate relation orderings:



Prune plans with cross-products immediately!

```
SELECT S.sname, B.bname, R.day  
FROM Sailors S, Reserves R, Boats B  
WHERE S.sid = R.sid AND R.bid = B.bid
```

## 2. Enumerate **join algorithm** choices:

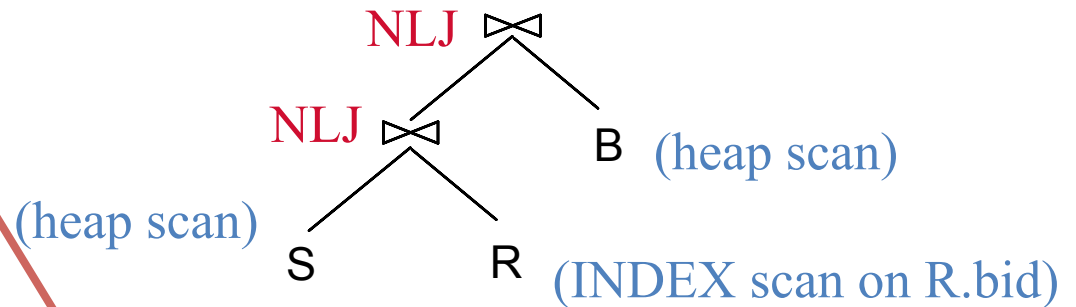
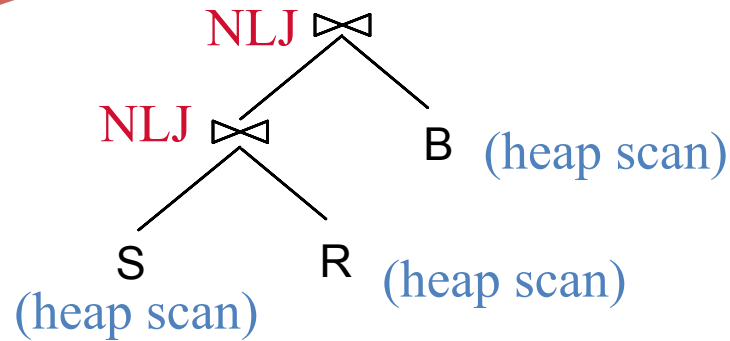
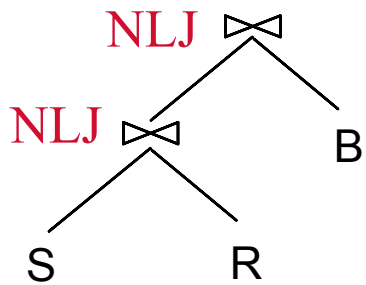


+ do same for  
4 other plans

→  $4 * 4 = 16$  plans so far..

```
SELECT S.sname, B.bname, R.day
FROM Sailors S, Reserves R, Boats B
WHERE S.sid = R.sid AND R.bid = B.bid
```

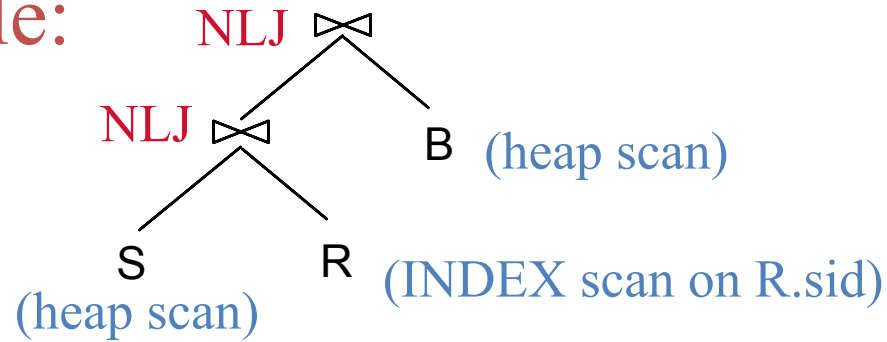
### 3. Enumerate access method choices:



+ do same for other plans

# Now estimate the cost of each plan

Example:





# Conclusions

- Ideas to remember:
  - canonical parse tree
  - syntactic q-opt – do selections/projections early
    - More complicated rules are also used
  - How to get selectivity estimations (uniformity, independence)
    - We saw mainly range and equality predicates
    - More complicated: histograms; join selectivity
  - left-deep joins
    - dynamic programming