(15) 1. What important conclusion follows from the following two facts? (1) The unbounded memory RAM can simulate any Turing machine. (2) There is a Turing machine which can simulate the computation of any unbounded memory RAM program.

(25) 2. Consider a language $L \in \mathbf{P}$, the TM M recognizing whether or not a string $w \in L$ in $p(|w|)$ steps, and some given string z. The computation of M on input z takes $T = p(|z|)$ steps, and can be simulated by a circuit $\mathcal{C}_{M,T}$. A description of this circuit $\mathcal{C}_{M,T}$ can be generated from M and z by a program $\mathcal{P}$. Explain exactly why the program $\mathcal{P}$ requires $\mathcal{O}(\log n)$ space to execute, where $n = |z|$.

(20) 3. A string w that reads the same left to right as right to left is called a *palindrome*. The language of palindromes, $L = \{w \mid w \in \Gamma^*,\ w = w^R\}$, over the alphabet Γ is not regular, and therefore is not accepted by a FSM. However, L is accepted by a nondeterministic pushdown automaton (PDA). Explain why L can *not* be accepted by a deterministic PDA.

(20) 4. In the proof that the language SATISFIABILITY (strings representing POSEs of Boolean functions that are 1 for some arguments) is **NP**-complete, a string describing a circuit (for CIRCUIT SAT) must be reduced to a string of clauses (a POSE for SATISFIABILITY). For instance, the instruction (i OR j k) for the OR gate taking inputs g_j and g_k and producing output $g_i = g_j \vee g_k$ must be reduced to an equivalent POSE in g_i, g_j, g_k. Give this POSE (the first clause is provided as a hint).

$(g_i \vee \bar{g}_j)$____________________________________

(10) 5. State the generalization of the pumping lemma for regular languages required to prove that $L = \{a^i b^j \mid i > j\}$ is not regular.

(10) 6. Which of the following languages are regular (answer yes/no)?

___ a) $\{w \in \Sigma^* \mid \text{length of } w \text{ is odd}\}$

___ b) $\{w \in \{a,b\}^* \mid w \text{ has } ab \text{ and } ba \text{ as substrings}\}$

___ c) $\{w \in \{a,b\}^* \mid w \text{ has twice as many } a\text{'s as } b\text{'s}\}$

___ d) $\{a^n b a^n \mid n \geq 0\}$

___ e) $\{w \mid w \text{ is decimal notation for an integer that is a multiple of } 5\}$