In this assignment you will try to interpolate historic USD/EUR foreign exchange data. The idea is that if your interpolation is clever, and you have managed to capture the key trends, you can make money by extrapolation into the future. That is if your interpolating/extrapolating function can predict the future USD/EUR rate, you can make money by either buying USD or selling it at the right time. You will be given historic data for a few past years, and you will use least square fitting to find the best functional dependence of USD/EUR as a function of time. You will then make a (virtual) currency purchase according to your predictions and see if you gain or lose on the transaction.

Student who has made most virtual ”money” will receive a REAL prize.

1 Background

As the cost of a currency changes, one can make money by buying it when it is cheap and selling it when it goes up in price (very complicated, isn’t it?). On the figure below you see a historical chart of EURO currency measured in US $ (USD). Simply put, if you buy at time \( t_1 \) and sell at time \( t_2 \),

\[
your \ \text{profit} = [R(t_2) - R(t_1) - C] \times M,
\]

where \( M \) is the amount of money you risk, \( R(t) \) is the exchange rate, \( C \) is a small commission that the clearing house takes per transaction. Assume \( C = 0.003 \) here. Also assume that your \( M = 100,000 \). Of course, your profit < 0 if \( R(t_2) < R(t_1) \). You can still make money, but you will need to sell at \( t_1 \) and buy at \( t_2 \). So, all you have to do is predict how \( R(t) \) will look like tomorrow, or 10 days later, or 3 month later.

Note that if you expect the future value of EUR to be lower, then you want to sell at \( t_1 \) and buy at \( t_2 \), in which case:

\[
your \ \text{profit} = [R(t_1) - R(t_2) - C] \times M,
\]

2 To Do. 30 points.

Basically, you need to find a meaningful fit function to the data you have, and use it to ”predict” the future. You will place your bets according to your prediction. So, perform either a linear or a non-linear least square fit (approximation) to the data set given.

The data set is given as a two dimensional column \( \{ t_k, y_k \} \), just like in Fig.1. I strongly suggest that you plot one of the data sets to see that for yourself. Also, get familiar with the fitting routines first, by generating some simple data sets and using fits to recover the input. Similar to what we did in class. For example, generate data that changes periodically with time, with some small noise, and fit it to a \( \sin[\cdot] \) function.
Figure 1: Historic chart of USD/EUR exchange rate. Cost of 1 EUR in USD. If you had bought and sold at points indicated, you would have made money.

Make sure you understand what you are doing. You must present these results in your printout.

3 Results

Results should be presented as a combined plot of the original data and your fit function; make sure you clearly indicate the interpolation region (past) and the extrapolation domain (future) by e.g. a vertical line. A mathematical equation for your fit function must be presented. The detailed buy/sell instructions should be specified (see below). You should also explain why, in your opinion, the non-liner fit algorithm did not always give you best fit.

Algorithm design/implementation.

There is really not much to do in terms of the code, just modify the least.square.FOREX.math shown in class. The key is to find best function, not to write a code. Look up least.square.nonlinear.math and least.square.FOREX.math on our class web-site to see what functions to use in Mathematica. Mathematica’s front-end is another useful source for syntax questions. You can read in an external file via data = ReadList["data", Number, Number]. Mathematica uses Levenberg-Marquardt method to do least square fits. This is also the default in Mathematica’s NonlinearModelFit[]. The method combines the steepest descent with a variation of a multi-dimensional Newton’s method for minimization. The details are not important for you here. But note that the method has all of the generic pitfalls of finding a global minimum in many dimensions.
4 The contest. 10 points.

Along with the best fit graph, functional form, and detailed reasoning of how you have arrived at it, you should also make a bet. That is you place all of your money $M = 100,000$ at the time $t_1 = 1195$ in present (this is the last data point available to you, EUR/USD = 1.2070) and predict a time $t_2 \leq 1500$ in the future where you it is best to sell (Or buy. You have to specify whether you sell or buy at $t_1$). Note that $t_1$ is fixed, you are not allowed to change it. After the homework has been collected, the "future" will be released, and you will compute how much money you have just made (or lost...), according to Eq. 1 if you your instructions were to buy first or Eq. 2 if you sell first. Of course, you can’t change your instructions past the deadline of this project. Bring these results to class once the “future” has been released. On the first page, near your name, you MUST give a short-hand for your prediction, in bold font. For example: “SELL 1316” or “BUY 1211”. The first instruction means that you ”buy at $t_1 = 1195$ and then sell at $t_2 = 1316$”; while the secon one means that you ”sell at $t_1 = 1195$ and then buy at $t_2 = 1211$.”