CS3414. Homework problem set II. **10 points per problem**, unless otherwise stated. Do not submit any codes, but be prepared to email one at TA/instructor’s request.

C&K = *Cheney and Kincaid* textbook. You can write your codes in C or C++. Do not submit the code unless asked for. Instead, outline your solution by showing the key steps in the algorithm used.

2. C&K 10, page 32.
3. Use Taylor series to show that the *truncation* error involved in calculating of the derivative \( f'(x) \approx (f(x+h/2) - f(x-h/2))/h \) is of order \( h^2 \), i.e. (truncation) error \( = A \times h^2 \).

3a. Use the above result, and the *round-off error* estimate discussed in class, to derive an expression for the *total* error involved in calculating \( f'(x) \). For simplicity, assume \( f'(x) \sim 1 \), along with its derivatives.

Find an estimate of the optimal step \( h \) that minimizes that error as a function of \( \epsilon_{mach} \). For your laptop (double precision), what is \( h \) and the associated total error? How does it compare with the optimal \( h \) and the error for the formula discussed in class, \( f'(x) \approx (f(x+h) - f(x))/h \)?

4. C&K 1, page 63

5. Write an efficient code that computes \( \exp(x) \) for any \(-25 < x < 25\) to within 3 decimal points. Provide printouts for \( x = 0.1, +20, -20 \). Clearly indicate which algorithms are used for different values of \( x \).

6. Write a code that produces accurate (within machine precision) values of \( f(x) = \frac{x - \sin(x)}{x^3} \) for \( 0 < x < 1 \). Print out results for \( x = 0.5, 10^{-16} \). Clearly indicate which algorithms are used for the two different values of \( x \). In this problem, you can use \( \sin(x) \) function supplied by standard libraries.

7. Use *Series* command in *Mathematica* to find first 6 terms in the Taylor series expansion of \( \cos(x) \). Given that, how would you compute \( \cos(10.0) \)? (10 rad).