FP Foundations, Scheme

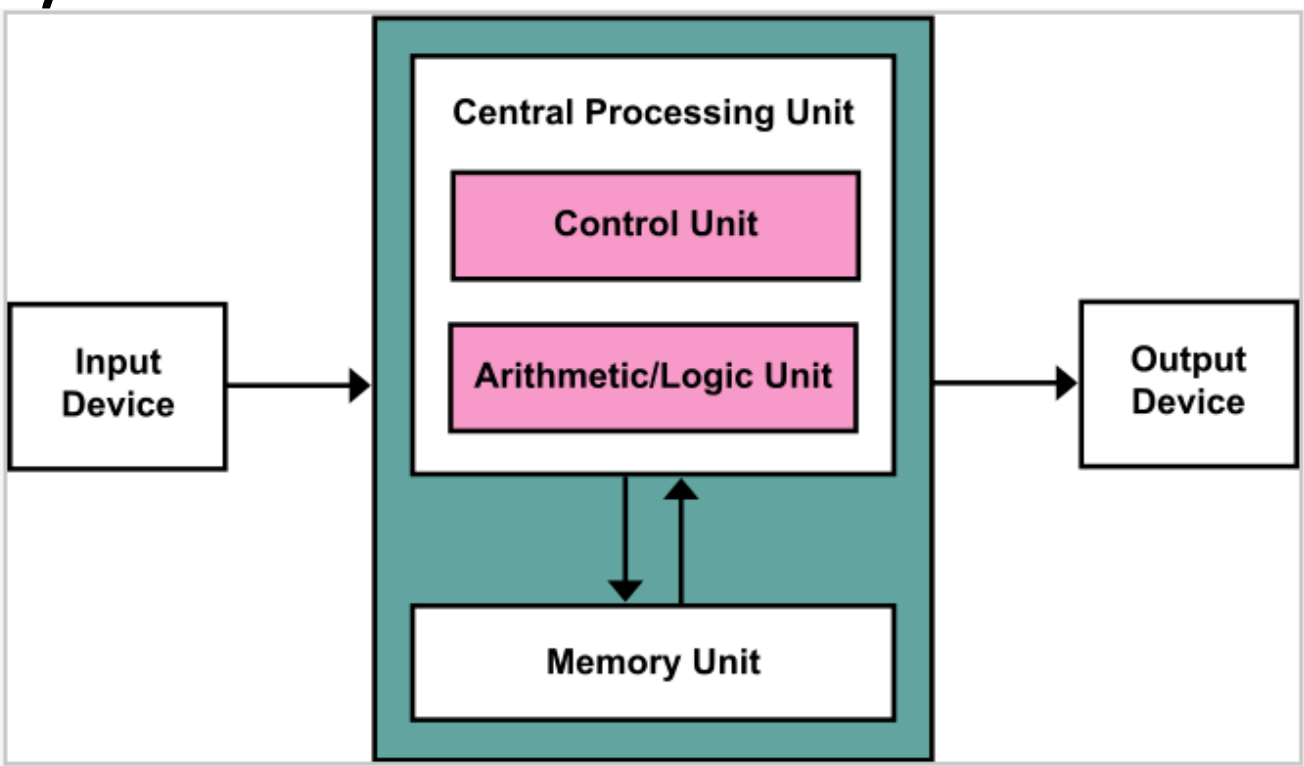
In Text: Chapter 15

Outline

- Mathematical foundations
- Functional programming
- λ-calculus
- LISP
- Scheme

Imperative Languages

- We have been discussing imperative languages
  - C/C++, Java, and Pascal are imperative languages
  - They follow the von Neumman architecture [1]



Functional Programming

- A different way of looking at things
  - It is based on mathematical functions
  - It is supported by functional and applicative programming languages
    - LISP, ML, Haskell

Mathematical Foundations

- A **mathematical function** is a mapping of members from one set to another set
  - The "input" set is called the domain
  - The "output" set is called the range

Mathematical Foundations

- The evaluation order of mapping expressions is controlled by recursion and conditional expressions, rather than by the sequencing and iterative repetition
- Functions do not have states
  - They have no side effects
  - They always produce the same output given the same input parameters

Simple Functions

- Usual form:
  function name + a list of parameters in parentheses + mapping expression
- E.g., cube(x) = x * x * x, where
  - both the domain and range sets are real numbers, and
  - x can represent any member of the domain set, but it is fixed to represent one specific element during the expression evaluation

Function Application

- It is specified by paring the function name with a particular element of the domain set
- The range element is obtained by evaluating the function-mapping expression with the domain element substituted for the particular element
  - Cube(2.0) = 2.0 * 2.0 * 2.0 = 8.0

Functional Forms

- A higher-order function, or **functional form**, is one that either takes functions as parameters, or yields a function as its result, or both
- Two common functional forms
  - Function composition
  - Apply-to-all

Function Composition

- **Function composition** has two functional parameters and yields a function whose value is the first function applied to the result of the second
- It is written as an expression, using a $\circ$ operator (called "circle" or "round")
  - E.g., $h = f \circ g$

    if $f(x) = x + 2$, and

    $g(x) = 3 * x$

    then $h(x) = f(g(x)) = (3 * x) + 2$

Apply-to-all

- **Apply-to-all** takes a single function as a parameter
- If applied to a list of arguments, apply-to-all applies its functional parameter to each element of the list, and then collects results in a list or sequence
- It is denoted by α
  - E.g., h(x) = x * x, then
    α(h, (2, 3, 4)) = (4, 9, 16)

Lambda expression

- Early theoretical work on functions separated the task of defining a function from that of naming the function
- Lambda notation, λ, provides a method for defining nameless functions
- A **lambda expression** is a function, which specifies the parameters, and the mapping expression
  - E.g., $\lambda(x)x * x * x$

Lambda-Calculus

- In the mid 1960s, Peter Landin observed that a complex programming language can be understood by formulating it as a tiny core calculus capturing the language's essential mechanisms, together with a collection of convenient derived forms whose behavior is understood by translating them into the core

Lambda-Calculus

- The core language used by Landin was the lambda-calculus, a formal system invented in the 1920s by Alonzo Church in which all computation is reduced to the basic operations of function definition and application

`factorial` Example

- factorial(n) =
  if n = 0 then 1 else n * factorial(n - 1)
- The corresponding λ-calculus term is:

  factorial(n) =

  λn. if n=0 then 1 else n * factorial(n - 1)
- Meaning
  - For each nonnegative number n, instantiating the function with the argument n yields the factorial of n as a result

λ-calculus

- Lambda-calculus embodies function definition and application in the purest possible form
- In the lambda-calculus, everything is a function
  - the arguments accepted by functions are themselves functions, and
  - the result returned by a function is another function

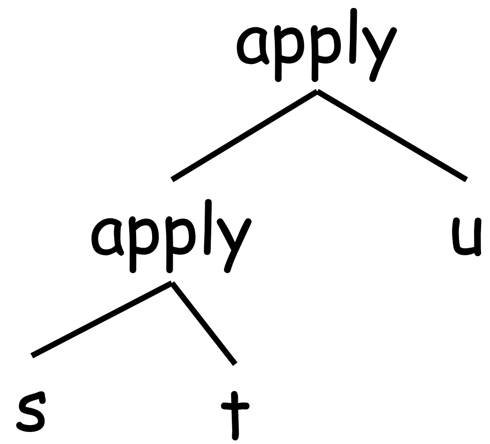
Syntax of λ-calculus

$$
\begin{aligned}
t ::= {} & x && \text{(a variable)} \\
\mid {} & \lambda x.t && \text{(a function)} \\
\mid {} & t\ t && \text{(function application)}
\end{aligned}
$$

- The syntax of lambda-calculus comprises three sorts of terms
  - Variable itself is a term
  - The abstraction of a variable x from a term t is a term
  - The application of term t_1 to another term t_2, is a term

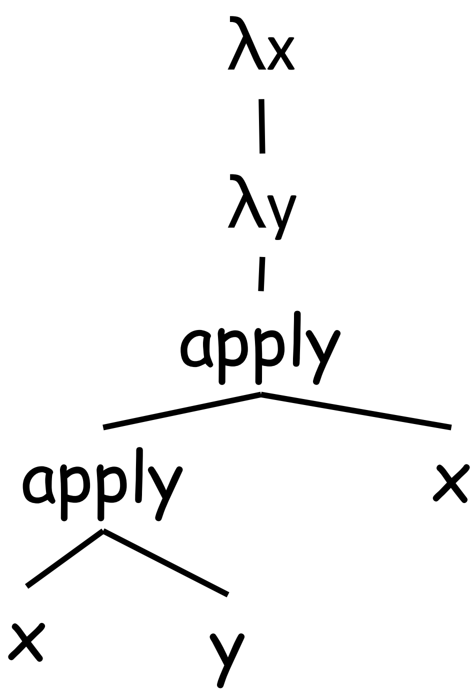
Two conventions of writing lambda-terms

- Application is left associative
  - Given s t u, the calculation is (s t) u



Two Conventions

- The body of abstraction is extended to right as much as possible
  - Given λx. λy. x y x, the calculation is λx. (λy. ((x y) x))



Scope

- An occurrence of the variable x is said to be bound when it occurs in the body t of an abstraction λx. t
- An occurrence of x is free if it appears in a position where it is not bound by an enclosing abstraction on x
  - In x y, and λy. x y, x is free
  - In λx. x, and λz. λx. λy. x (y z), x is bound

Scope (cont'd)

- A term with no free variable is said to be closed
- Closed terms are also called combinators
- The simplest combinator is called the identity function:
  id = λx. x

Operational Semantics

- $(\lambda x.\ t_{12})t_2 \rightarrow (x \mapsto t_2)t_{12}$
  - Evaluate the term t_{12} by replacing every occurrence of x with t_2
  - What is the reduction result of $(\lambda x.\ x)\ y$?
  - What is the evaluation result of the term $(\lambda x.\ x\ (\lambda x.\ x))(u\ r)$?
  - All terms of the form $(\lambda x.\ t_{12})t_2$ is called redex (reducible expression)
  - The operation of rewriting a redex according to the above rule is called beta-reduction

An Example of Reduction

- $(\lambda x.\ x)\ ((\lambda x.\ x)(\lambda z.\ (\lambda x.\ x)\ z))$

-> $(\lambda x.\ x)(\lambda z.\ (\lambda x.\ x)\ z)$

-> $\lambda z.\ (\lambda x.\ x)\ z$

-> $\lambda z.\ z$

Programming in the Lambda-Calculus

- Multiple arguments
  - Lambda-calculus provides no built-in support for multi-argument functions
  - But we can use higher-order functions to achieve the same effect

Multiple Arguments

- Suppose
  - s is a term involving two free variables x and y
  - We want to write a function f, such that for each pair of arguments (v, w), f yields the result of substituting v for x, and w for y
  - f = λx. λy. s
  - Applying f to (v, w): f v w

Multiple Arguments

- The transformation of multi-argument functions into higher-order functions is called ***currying***