# Polymorphic Types

**Polymorphic functions are achieved using type variables.**

- A **type variable** starts with a prime `'`.
  - `'a` read $\alpha$
  - `'b` read $\beta$

- Examples:

```
- fun swap(x,y)=(y,x);
val swap = fn : 'a * 'b -> 'b * 'a

- fun map f [] = []
=    | map f (head::tail) =
=           (f head)::(map f tail);
val map =
  fn : ('a -> 'b) -> 'a list -> 'b list

- map swap [(3,"c"),(5,"f"),(17,"z")];
val it = [("c",3),("f",5),("z",17)] :
         (string * int) list
```

# Composition

```
- infix o;
infix o
- fun (f o g) x = f (g x);
val o =
  fn : ('a -> 'b) * ('c -> 'a) -> 'c -> 'b

- fun plus3 x = x + 3; fun times4 y = y * 4;
val plus3 = fn : int -> int
val times4 = fn : int -> int

- fun times4plus3 z = (plus3 o times4) z;
val times4plus3 = fn : int -> int

- times4plus3 5;
val it = 23 : int
```

# Equality Types

- Type variables for data types that must support equality begin with ''.
  - ''a read $\alpha^=$
  - ''b read $\beta^=$

- Example — list membership:

```
- fun member (e,[]) = false
=    | member(e,(head::tail)) =
=       (e=head) orelse (member (e,tail));
val member = fn : ''a * ''a list -> bool

- member ("t",["a","b","c"]);
val it = false : bool

- member (false,[true,true,false,true]);
val it = true : bool
```

# Heterogeneous Types

The `datatype` declaration handles heterogeneous data:

```
- type point = real * real;
type point = real * real
- datatype shape =
=     Circle of point * real
=       (* (center,radius) *)
=   | Triangle of point * point * point
=       (* 3 points *)
=   | Square of point * point * point *
=       point  (* 4 points *);
datatype shape
  = Circle of (real * real) * real
  | Square of (real * real) *
       (real * real) * (real * real) *
       (real * real)
  | Triangle of (real * real) *
       (real * real) * (real * real)
```

# Heterogeneous Types

- ```
  - Circle;
  val it = fn : point * real -> shape

  - Square;
  val it = fn : point * point *
                point * point -> shape

  - Triangle;
  val it = fn : point * point *
                point -> shape
  ```

- `Circle` is a function that takes a point and a radius and returns an object of type `shape`.

  ```
  - Circle ( (1.0,2.5), 0.75);
  val it = Circle ((1.0,2.5),0.75) : shape
  ```
  ```

# Enumeration Types

**An enumeration type consists of a finite number of constants.**

```
- datatype bool = true | false;
datatype bool = false | true

- false;
val it = false : bool

- datatype color = Red | Yellow | Blue |
=                  Green;
datatype color = Blue | Green | Red | Yellow

- Green;
val it = Green : color
```

# Function on Shapes

**Use pattern matching on datatypes.**

```
- fun center (Circle(c,r)) = c
=   | center (Triangle((x1,y1),(x2,y2),
=                (x3,y3))) =
=              ((x1+x2+x3)/3.0,(y1+y2+y3)/3.0)
=   | center (Square((x1,y1),(x2,y2),
=                (x3,y3),(x4,y4))) =
=              ((x1+x2+x3+x4)/4.0,
=               (y1+y2+y3+y4)/4.0);
val center = fn : shape -> point

- center (Circle((2.5,3.78),4.97));
val it = (2.5,3.78) : point

- center (Square((0.0,0.0),(1.0,0.0),
=                  (0.0,1.0),(1.0,1.0)));
val it = (0.5,0.5) : point
```

# Binary Trees

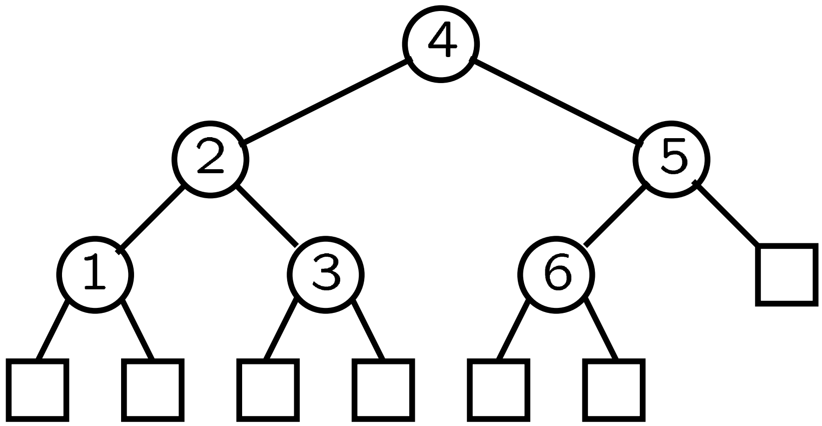
```
- datatype 'a tree = Lf
=   | Br of 'a * 'a tree * 'a tree;
datatype 'a tree = Br of 'a * 'a tree *
  'a tree | Lf

- fun size Lf = 0
=   | size (Br(v,t1,t2)) =
        1 + size t1 + size t2;
val size = fn : 'a tree -> int

- fun depth Lf = 0
=   | depth (Br(v,t1,t2)) =
        1 + Int.max (depth t1, depth t2);
val depth = fn : 'a tree -> int

- fun preord (Lf, vs) = vs
=   | preord (Br(v,t1,t2), vs) =
=     v :: preord (t1, preord (t2, vs));
val preord =
  fn : 'a tree * 'a list -> 'a list
```

# Binary Trees Continued

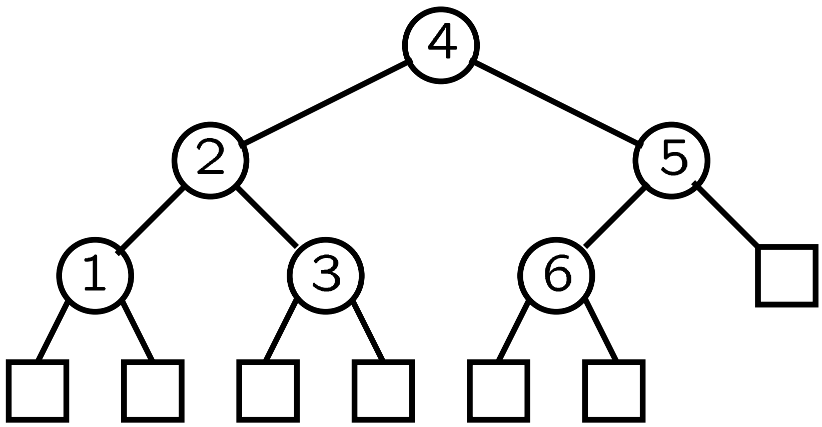



```
- val tree2 = Br(2, Br(1,Lf,Lf),
                    Br(3,Lf,Lf));
val tree2 = Br (2,Br (1,Lf,Lf),
                  Br (3,Lf,Lf)) : int tree

- val tree5 = Br(5, Br(6,Lf,Lf), Lf);
val tree5 = Br (5,Br (6,Lf,Lf),Lf) :
  int tree

- val tree4 = Br(4, tree2, tree5);
val tree4 = Br (4,Br (2,Br #,Br #),
                  Br (5,Br #,Lf)) : int tree
```

# Binary Trees Concluded




```
- size tree4;
val it = 6 : int

- depth tree4;
val it = 3 : int

- preord (tree4,[]);
val it = [4,2,1,3,5,6] : int list
```

# Graphs

**A directed graph can be represented as a list of ordered pairs.**

```
- datatype 'node graph =
    Graph of ('node * 'node) list;
datatype 'a graph = Graph of ('a * 'a) list

- val beats = Graph [("paper","rock"),
    ("rock","scissors"),
    ("scissors","paper")];
val beats = Graph [("paper","rock"),
    ("rock","scissors"),
    ("scissors","paper")]
  : string graph

- val divides =
    Graph [(3,6),(3,12),(6,12),
           (3,21),(6,18),(3,18)];
val divides =
    Graph [(3,6),(3,12),(6,12),
           (3,21),(6,18),(3,18)] : int graph
```

# Basic Graph Functions

- Successor:

```
- fun nexts (a, Graph []) = []
=   | nexts (a, Graph ((x,y) :: rest)) =
=       if a=x then
                  y :: nexts(a,Graph rest)
=              else nexts(a,Graph rest);
val nexts =
  fn : ''a * ''a graph -> ''a list

- nexts (3, divides);
val it = [6,12,21,18] : int list
```

- Nodes:

```
- fun nodes (Graph x) =
=   (unique o flatten) x;
val nodes = fn : ''a graph -> ''a list

- nodes divides;
GC #0.0.0.0.3.144:   (4 ms)
val it = [12,21,6,3,18] : int list
```

# Flatten and Unique

- Flatten:

```
- fun flatten [] = []
=   | flatten ((x,y)::tail) =
=       x :: y :: flatten tail;
val flatten =
  fn : ('a * 'a) list -> 'a list
```

- Unique:

```
- fun member (e,[]) = false
=   | member(e,(head::tail)) =
=     (e=head) orelse (member (e,tail));
val member = fn : ''a * ''a list -> bool

- fun unique [] = []
=   | unique (head :: tail) =
=       if member(head,tail)
=         then unique tail
=         else head :: unique tail;
val unique = fn : ''a list -> ''a list
```

# Depth-First Search

**Use depth-first search to determine the nodes reachable from a particular node.**

```
- fun depthf ([], Graph graph, visited) =
=       rev visited
=   | depthf (head::tail, Graph graph,
=             visited) =
=     if member(head,visited)
=     then depthf (tail,Graph graph,visited)
=     else depthf
=         (nexts(head, Graph graph) @ tail,
=          Graph graph, head::visited);
val depthf =
  fn : ''a list * ''a graph * ''a list ->
       ''a list

- depthf ([3],divides,[]);
val it = [3,6,12,18,21] : int list

- depthf ([6],divides,[]);
val it = [6,12,18] : int list
```

# Depth-First Search

## Faster version. Avoids append.

```
- fun depth ([], Graph graph, visited) =
=       rev visited
=   | depth (head::tail, Graph graph,
=             visited) =
=       depth (tail, Graph graph,
=       if member(head,visited) then visited
=       else depth (nexts(head,Graph graph),
=             Graph graph, head::visited));
val depth =
  fn : ''a list * ''a graph * ''a list ->
       ''a list

- depth ([6],divides,[]);
val it = [12,6,18] : int

- depth (["rock"], beats, []);
val it = ["paper","scissors","rock"] :
  string list
```

# Exceptions

- Declaring exceptions:

  **`exception`** *exid*

  or

  **`exception`** *exid* **`of`** *type*

- Raising exceptions:

  **`raise`** *exid*

- Handling exceptions:

  *exp* **`handle`** $pat_1$ `=>` $exp_1 | \cdots |$ $pat_n$ `=>` $exp_n$

# Exception Example

```
- exception OddOne;
exception OddOne

- fun sum_even [] = 0
=   | sum_even (head::tail) =
=     if (head mod 2) = 1 then raise OddOne
=     else head + (sum_even tail);
val sum_even = fn : int list -> int

- val t = [2,4,6,8,12,13,14,17,20];
val t = [2,4,6,8,12,13,14,17,20] : int list

- sum_even t;
GC #0.0.0.0.2.28:   (2 ms)

uncaught exception OddOne
  raised at: stdIn:39.36-39.42

- sum_even t handle OddOne => ~1
                  | Any => ~3;
val it = ~1 : int
```

# Infinite Data

**An infinite list (called a sequence or lazy list) is an ordered pair where the second item is a function to generate the rest of the list.**

```
- datatype 'a seq = Nil
=         | Cons of 'a * (unit -> 'a seq);
datatype 'a seq =
  Cons of 'a * (unit -> 'a seq) | Nil

- fun from n = Cons(n, fn() => from (n+1));
val from = fn : int -> int seq

- from 0;
val it = Cons (0,fn) : int seq
```

# Sequences Continued

```
- fun power_2 n =
=   Cons(n, fn () => power_2 (2*n));
val power_2 = fn : int -> int seq

- val p = power_2 1;
val p = Cons (1,fn) : int seq

- fun takeq (0,sequence) = []
=   | takeq (n, Nil) = []
=   | takeq (n, Cons(head,tail)) =
=     head :: takeq(n-1, tail ());
val takeq = fn : int * 'a seq -> 'a list

- takeq (10,p);
val it = [1,2,4,8,16,32,64,128,256,512] :
         int list
```

# Using a Sequence

```
- fun squares Nil : int seq = Nil
=   | squares (Cons(head,tail)) =
=     Cons(head*head,
=          fn() => squares (tail()));
val squares = fn : int seq -> int seq

- val sq = squares (from 1);
val sq = Cons (1,fn) : int seq

- takeq (12,sq);
val it =
  [1,4,9,16,25,36,49,64,81,100,121,144] :
  int list
```

# The Sequence of Primes

**Start with a function to filter a sequence according to a predicate.**

```
- fun filterq pred Nil = Nil
=   | filterq pred (Cons(head,tail)) =
=     if pred head then
=     Cons(head,
=       fn() => filterq pred (tail()))
=     else filterq pred (tail());
val filterq =
  fn : ('a -> bool) -> 'a seq -> 'a seq
```

**Now build a predicate to sift a sequence of integers by division by a prime.**

```
- fun sift p =
=   filterq (fn n => n mod p <> 0);
val sift = fn : int -> int seq -> int seq
```

# The Sequence of Primes Continued

**Finally construct a function that sifts by all primes.**

```
- fun sieve Nil = Nil
=   | sieve (Cons(p,rest)) =
=   Cons(p,
=     fn () => sieve(sift p (rest())));
val sieve = fn : int seq -> int seq
```

**Create the sequence of primes.**

```
- val primes = sieve (from 2);
val primes = Cons (2,fn) : int seq

- takeq(15,primes);
val it =
  [2,3,5,7,11,13,17,19,23,29,31,37,...] :
  int list
```

# Reference Semantics

- A reference to an object in the heap:

```
- val l = ref primes;
val l = ref (Cons (2,fn)) : int seq ref
```

- The object that is referenced:

```
- !l;
val it = Cons (2,fn) : int seq
```

- An assignment:

```
- l := sq;
val it = () : unit
- !l;
val it = Cons (1,fn) : int seq
```

- Assignment requires **type match.**

# Structures

- Abstract data types or modules in ML are called **structures.**

- A structure encapsulates types, values, and functions.

- External access to these can be controlled by **signatures.**

# Stack Structure

```
signature STACK =
  sig
    type element
    val pop : unit -> element
    val push : element -> unit
    val empty : unit -> bool
  end

structure Stack : STACK =
  struct
    type element = int
    val stack = ref [] : element list ref
    fun pop () =
      case !stack of
        [] => ~1
      | (top::bottom) =>
          (stack := bottom; top)
    fun push x =
      (stack := x::(!stack))
    fun empty () = (!stack = [])
  end;
```

# Use of Stack

```
- use "Stack.sml";
[opening Stack.sml]
signature STACK =
  sig
    type element
    val pop : unit -> element
    val push : element -> unit
    val empty : unit -> bool
  end
structure Stack : STACK
val it = () : unit

- Stack.empty();
val it = true : bool
```

# Use of Stack Continued

```
- Stack.push 5; Stack.push 12; Stack.push 9;
val it = () : unit
val it = () : unit
val it = () : unit

- Stack.empty();
val it = false : bool

- Stack.pop(); Stack.pop();
= Stack.pop(); Stack.pop();
val it = 9 : Stack.element
val it = 12 : Stack.element
val it = 5 : Stack.element
val it = ~1 : Stack.element

- Stack.stack;
stdIn:18.1-18.12
  Error: unbound variable or constructor:
  stack in path Stack.stack
```

# Opening the Stack

```
- open Stack;
opening Stack
  type element = int
  val pop : unit -> element
  val push : element -> unit
  val empty : unit -> bool

- empty();
val it = true : bool

- stack;
stdIn:16.1-16.6
  Error: unbound variable or constructor:
  stack
```

# Opening the Stack Continued

```
- push 44; push 112; push 97;
val it = () : unit
val it = () : unit
val it = () : unit

- empty();
val it = false : bool

- pop(); pop();
val it = 97 : element
val it = 112 : element

- Stack.empty();
val it = false : bool

- fun pair (x: element) = (x,x);
val pair = fn : element -> element * element
```

# ML Summary

- A strongly-typed, functional programming language with type inferencing
- Interpreted or compiled
- Functions are first-class objects. Function application is the essence of computation.
- Cases are handled by pattern matching.
- Supports polymorphic types
- Supports infinite data structures, but has eager evaluation
- Supports exceptions, type-safe imperative operations, and abstract data structures