FP Foundations, Scheme

In Text: Chapter 14

Mathematical Functions

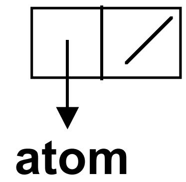
- Def: A mathematical function is a mapping of members of one set to another set
- The "input" set is called the domain
- The "output" set is called the range
- Each input maps to exactly one output

Scheme Syntax Basics

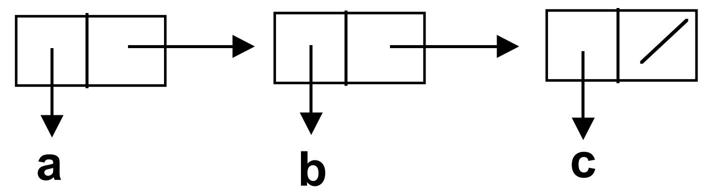
- Case-insensitive
- Data Types:
  - Atoms: identifiers, symbols, numbers
  - Lists (S-expressions)
    - List form: parenthesized collections of sublists and/or atoms
    - (a b c d)
    - (a (b c) d e)
- All lists are internally represented by singly-linked chains where each node has 2 pointers (think “data” and “next”)

Interal List Representation

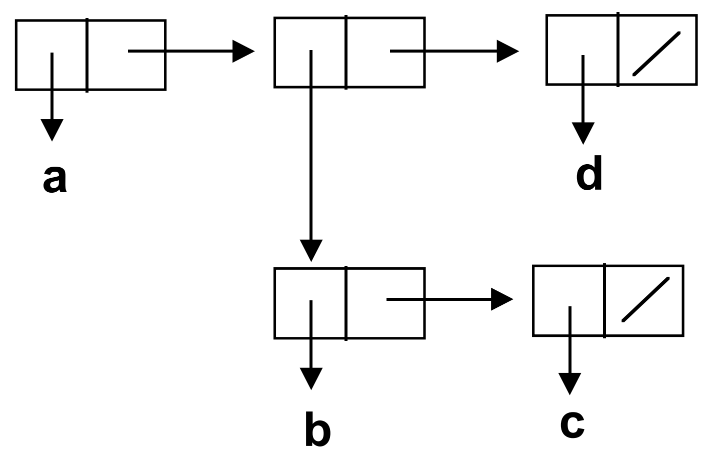
- Single atom:



- List of atoms: (a b c)



- List containing list:
  (a (b c) d)



Primitive Functions

1. Arithmetic: +, -, *, /, abs, sqrt

```
(+ 5 2)
(* 47 (+ (- 5 3) 2))
```

2. QUOTE takes one parameter; returns the parameter without evaluation

- Parameters to a function are evaluated before applying the function; use QUOTE to prevent it when inappropriate
- QUOTE can be abbreviated with the apostrophe prefix operator

```
'(a b)
(quote (a b))
```

Primitive Functions (cont.)

3. CAR takes a list parameter; returns the first element of that list

```
(car '(a b c))       yields   a
(car '((a b) c d))   yields   (a b)
```

4. CDR takes a list parameter; returns the list after removing its first element

```
(cdr '(a b c))       yields   (b c)
(cdr '((a b) c d))   yields   (c d)
```

Primitive Functions (cont.)

5. CONS takes two parameters; returns a new list that includes the first parameter as its first element and the second parameter as the remainder

```
(cons 'a '(b c))    returns    (a b c)
```

6. LIST takes any number of parameters; returns a list with the parameters as elements

```
(list 'a '(b c))    returns    (a (b c))
```

Predicate Functions

- A predicate is a function returning a boolean value
- #T is true and (), the empty list, is false
- By convention, Scheme predicates have names that end in a "?"

1. EQUAL? takes two parameters; it returns #T if both parameters are "the same"; works on about everything

Predicate Functions (cont.)

2. EQV? (atom equality) takes two symbolic parameters; it returns #T if both parameters are atoms and the two are the same

(eqv? 'a 'a) yields #t

(eqv? 'a '(a b)) yields ()

- EQV? is unreliable if used on lists

3. EQ? (pointer equality) is like EQV?, but is also unreliable on numbers, characters, and a few other special cases

Predicate Functions (cont.)

4. LIST? takes one parameter; it returns #T if the parameter is an list; otherwise ()

5. NULL? takes one parameter; it returns #T if the parameter is the empty list; otherwise ()

6. Numeric Predicate Functions: =, >, <, >=, <=, even?, odd?, zero?, positive?, negative?

Other Useful Functions

- (write expression)
- (write-string expression)
- (newline)
- (read-string stop-character-set)

Lambda Expressions

- A lambda expression describes a "nameless" function
- Specifies both the parameter(s) and the mapping
- Consider this function cube (x) = x * x * x
- Corresponding lambda expr: ?(x) x * x * x
- Can be "applied" to parameter(s) by placing the parameter(s) after the expression

  e.g. (?(x) x * x * x)(3)
- The above application evaluates to 27

Lambda Expressions in Scheme

- Based on lambda notation

```
(lambda (l) (car (car l)))
```

- L is called a “bound variable”; think of it as a formal parameter name
- Lambda expressions can be applied

```
((lambda (l) (car (car l))) '((a b) c d))
```

A Function for Constructing Functions

DEFINE has two forms:

1. To bind a symbol to an expression:

```
(define pi 3.141593)
(define two_pi (* 2 pi))
```

2. To bind names to lambda expressions

```
(define (cube x)
    (* x x x)
)
```

Example use: (cube 4)

Functional Forms

- Def: A higher-order function, or functional form, is one that:
  - Takes function(s) as parameter(s), or
  - Yields a function as its result, or
  - Both

Function Composition

- A functional form that takes two functions as parameters and yields a function whose result is a function whose value is the first actual parameter function applied to the result of the application of the second
- Form: $h \equiv f \circ g$
- Which means: $h(x) \equiv f(g(x))$

Construction

- A functional form that takes a list of functions as parameters and yields a list of the results of applying each of its parameter functions to a given parameter
- Form: [f, g]
- For f (x) ≡ x * x * x and g (x) ≡ x + 3,

  [f, g] (4) yields (64, 7)

Apply-to-All

- A functional form that takes a single function as a parameter and yields a list of values obtained by applying the given function to each element of a list of parameters
- Form: α
- For $h(x) \equiv x * x * x$
- $\alpha(h, (3, 2, 4))$ yields $(27, 8, 64)$
- "map" in Scheme is a very flexible version of this

Scheme Evaluation Process

1. Parameters are evaluated, in no particular order
2. The values of the parameters are substituted into the function body
3. The function body is evaluated
4. The value of the last expression in the body is the value of the function

- (Special forms use a different evaluation process)

Control Flow: Selection

1. Selection: the special form, IF

(IF *predicate then_exp else_exp*)

```
(if (not (zero? count ))
    (/ sum count)
    0
)
```

Control Flow: Multi-Way Selection

2. Multiple selection: the special form, COND

```
(COND
  (predicate_1  expr {expr})
  (predicate_1  expr {expr})
  ...
  (predicate_1  expr {expr})
  (ELSE expr {expr})
)
```

- Returns the value of the last *expr* in the first pair whose *predicate* evaluates to true

Introducing Locals: Let

3. Internal definitions: the special form, LET

```
(let (  (x '(a b c))
        (y '(d e f))   )
   (cons x y)
)
```

- Introduces a list of local names (use define for top-level entities, but use let for internal definitions)
- Each name is given a value
- Use let* if later values depend on earlier ones

Control Flow: Named Let

2. Recursive-style looping: the special form, named LET

(COND
  (*predicate_1 expr* {*expr*})
  (*predicate_1 expr* {*expr*})
  ...
  (*predicate_1 expr* {*expr*})
  (ELSE *expr* {*expr*})
)

- Returns the value of the last *expr* in the first pair whose *predicate* evaluates to true

Example Scheme Functions: member

- member takes an atom and a list; returns #T if the atom is in the list; () otherwise

```
(define (member atm lis)
  (cond
    ((null? lis) '())
    ((equal? atm (car lis)) #t)
    (else (member atm (cdr lis)))
  )
)
```

Example Scheme Fns: flat-equal

- flat-equal takes two simple lists as parameters; returns #T if the two simple lists are equal; () otherwise

```
(define (flat-equal lis1 lis2)
   (cond
      ((null? lis1) (null? lis2))
      ((null? lis2) '())
      ((eqv? (car lis1) (car lis2))
          (flat-equal (cdr lis1) (cdr lis2)))
      (else '())
   )
)
```

Example Scheme Fns: equal

- equal takes two lists as parameters; returns #T if the two general lists are equal; () otherwise

```
(define (equal lis1 lis2)
   (cond
      ((not (list? lis1))   (eqv? lis1 lis2))
      ((not (list? lis2))   '())
      ((null? lis1)         (null? lis2))
      ((null? lis2)         '())
      ((equal (car lis1) (car lis2))
                            (equal (cdr lis1) (cdr lis2)))
      (else                 '())
   )
)
```

Example Scheme Fns: append

- append takes two lists as parameters; returns the first parameter list with the elements of the second parameter list appended at the end

```
(define (append lis1 lis2)
   (cond
      ((null? lis1) lis2)
      (else
         (cons
            (car lis1)
            (append (cdr lis1) lis2)))
   )
)
```

Creating Functional Forms

- Composition: the previous examples have used it
- Apply to All: one form in Scheme is mapcar
- Applies the given function to all elements of the given list; result is a list of the results

```
(define (mapcar fun lis)
  (cond
    ((null? lis) '())
    (else (cons (fun (car lis))
                (mapcar fun (cdr lis))))
  ))
```

Interpretive features

- One can define a function that builds Scheme code and requests its interpretation
- The interpreter is a user-available function, EVAL
- Suppose we have a list of numbers that must be added together

```
(define (adder lis)
   (cond
      ((null? lis) 0)
      (else (eval (cons '+ lis)))
   ))
```

- The parameter is a list of numbers to be added; adder inserts a + operator and interprets the resulting list

Imperative Features in Scheme

- "Functions" that modify/change a data structure are called mutators
- By convention, mutator names end in "!"
- SET! binds or rebinds a value to a name
- SET-CAR! replaces the car of a list
- SET-CDR! replaces the cdr part of a list

Common LISP

- A combination of many of the features of the popular dialects of LISP around in the early 1980s
- A large and complex language--the opposite of Scheme
- Includes:
  - records
  - arrays
  - complex numbers
  - character strings
  - powerful i/o capabilities
  - packages with access control
  - imperative features like those of Scheme
  - iterative control statements

Standard ML

- A static-scoped functional language with syntax that is closer to Pascal than to LISP
- Uses type declarations, but also does type inferencing to determine the types of undeclared variables (See Chapter 4)
- Strongly typed (whereas Scheme has latent typing) and has no type coercions
- Includes exception handling and a module facility for implementing abstract data types
- Includes lists and list operations
- The val statement binds a name to a value (similar to DEFINE in Scheme)

SML Function Declarations

```
fun function_name (formal_parameters) =
        function_body_expression;

fun cube (x : int) = x * x * x;
```

- List-based operations can be polymorphic, using type inferencing
- Functions that use arithmetic or relational operators cannot be polymorphic

Haskell

- Similar to ML
  - syntax, static scoped, strongly typed, type inferencing
- Different from ML (and most other functional languages) in that it is PURELY functional
  - no variables, no assignment statements, and no side effects of any kind
- Most Important Features
  - Uses lazy evaluation (evaluate no subexpression until the value is needed)
  - Has “list comprehensions,” which allow it to deal with infinite lists