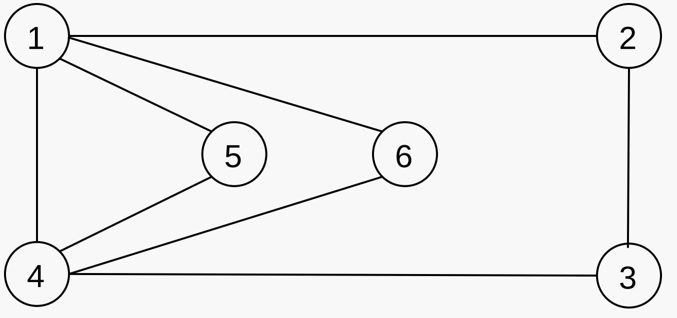
Euler Circuit

A path through a graph is an *Euler Circuit* if it has the following two properties:

- the path traverses every edge in the graph, exactly once
- the path ends at the same vertex where it starts



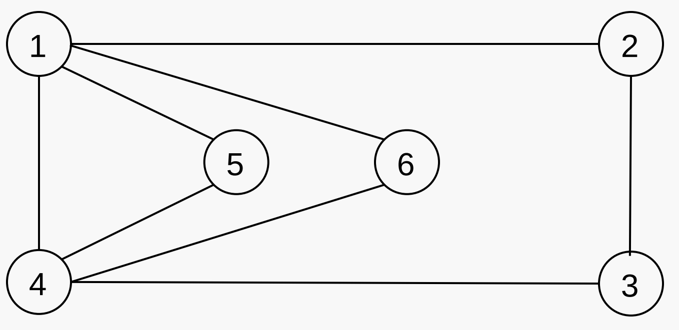
Under what condition(s) is a graph guaranteed to have an Euler Circuit?

... iff every vertex has an even number of edges

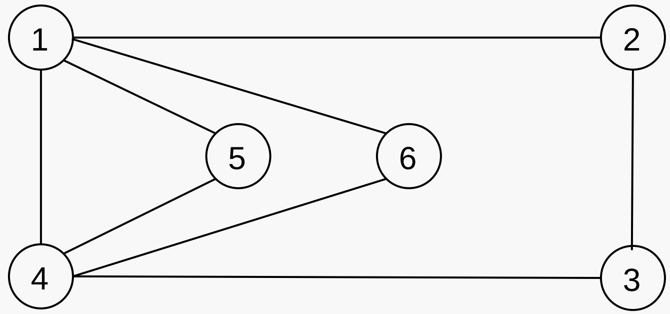
Computing an Euler Circuit

To compute an Euler Circuit, or show there is not one:

- pick a vertex and perform a depth-first traversal, marking edges that are traversed, and marking any vertex that is reached and has no remaining untraversed edges
- if this does not lead to a circuit, there is no Euler Circuit in the graph
- pick a vertex with an untraversed edge, and repeat the first step



Computing an Euler Circuit



Pick vertex 1:

V	Edges			
1	2	4	5	6
2	1	3		
3	2	4		
4	1	3	5	6
5	1	4		
6	1	4		

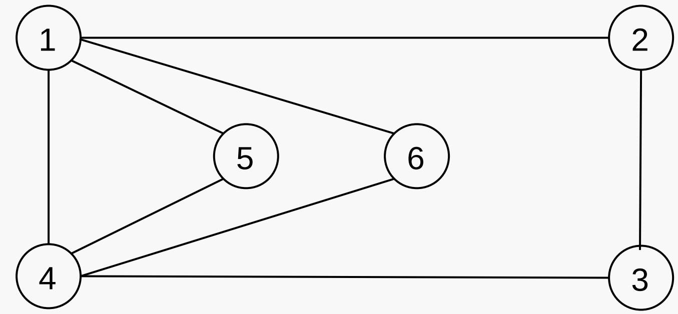
V	Edges			
1	~~2~~	4	5	6
2	~~1~~	3		
3	2	4		
4	1	3	5	6
5	1	4		
6	1	4		

Path1: 1-2

V	Edges			
1	~~2~~	4	5	6
~~2~~	~~1~~	~~3~~		
3	~~3~~	4		
4	1	3	5	6
5	1	4		
6	1	4		

Path1: 1-2-3

Computing an Euler Circuit



V	Edges			
1	~~2~~	4	5	6
~~2~~	~~1~~	~~3~~		
~~3~~	~~3~~	~~4~~		
4	1	~~3~~	5	6
5	1	4		
6	1	4		

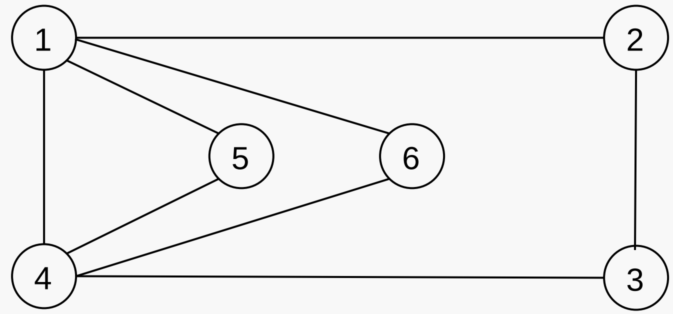
Path1: 1-2-3-4

V	Edges			
1	~~2~~	~~4~~	5	6
~~2~~	~~1~~	~~3~~		
~~3~~	~~3~~	~~4~~		
4	~~1~~	~~3~~	5	6
5	1	4		
6	1	4		

Path1: 1-2-3-4-1

Computing an Euler Circuit

Pick vertex 1 again:



V	Edges			
1	~~2~~	~~4~~	~~5~~	6
~~2~~	~~1~~	~~3~~		
~~3~~	~~3~~	~~4~~		
4	~~1~~	~~3~~	5	6
5	~~1~~	4		
6	1	4		

Path2: 1-5

V	Edges			
1	~~2~~	~~4~~	~~5~~	6
~~2~~	~~1~~	~~3~~		
~~3~~	~~3~~	~~4~~		
4	~~1~~	~~3~~	~~5~~	6
~~5~~	~~1~~	~~4~~		
6	1	4		

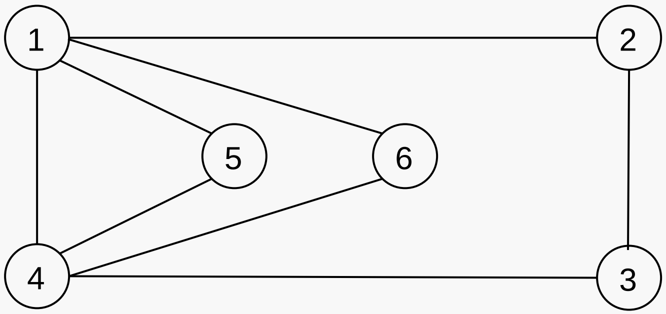
Path2: 1-5-4

V	Edges			
1	~~2~~	~~4~~	~~5~~	6
~~2~~	~~1~~	~~3~~		
~~3~~	~~3~~	~~4~~		
4	~~1~~	~~3~~	~~5~~	~~6~~
~~5~~	~~1~~	~~4~~		
6	1	~~4~~		

Path2: 1-5-4-6

Computing an Euler Circuit

V	Edges			
~~1~~	~~2~~	~~4~~	~~5~~	~~6~~
~~2~~	~~1~~	~~3~~		
~~3~~	~~3~~	~~4~~		
4	~~1~~	~~3~~	~~5~~	~~6~~
~~5~~	~~1~~	~~4~~		
~~6~~	~~1~~	~~4~~		



Path2: 1-5-4-6-1

Path1: 1-2-3-4-1

Path2: 1-5-4-6-1

Path: 1-5-4-6-1-2-3-4-1

$$\Theta\left(\left|V\right|+\left|E\right|\right)$$