You may work in pairs for this assignment. If you choose to work with a partner, make sure only one of you submits a solution, and you paste a copy of the Partners Template that contains the names and PIDs of both students at the beginning of the file you submit.

You will submit your solution to this assignment to the Curator System (as HWO2). Your solution must be either a plain text file (e.g., NotePad++) or a typed MS Word document; submissions in other formats will not be graded.

Partial credit will only be given if you show relevant work.

## Part I

1. Consider a hash table consisting of $M=11$ slots, and suppose nonnegative integer key values are hashed into the table using the hash function h1 ():
```
public static int h1( int key ) {
    int x = (key + 7) * (key + 13);
    x = x >> 4;
    x = x + key;
    return x;
}
```

a) [27 points] Suppose that collisions are resolved by using quadratic probing, with the probe function:

$$
\left(k^{2}+k\right) / 2
$$

The integer key values listed below are to be inserted, in the order given. Show the home slot (the slot to which the key hashes, before any probing), the probe sequence (if any) for each key, and the final contents of the hash table after the following key values have been inserted in the given order:

| Key <br> Value | Home <br> Slot | Probe Sequence |
| :---: | :---: | :---: |
| 43 |  |  |
| 23 |  |  |
| 15 |  |  |
| 27 |  |  |
| 31 |  |  |
| 14 |  |  |
| 37 |  |  |
| 21 |  |  |
| 29 |  |  |

## Final Hash Table:

| Slot | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Contents |  |  |  |  |  |  |  |  |  |  |  |

b) [27 points] Suppose that collisions are resolved by using double hashing (see the course notes), with the secondary hash function Reverse (key), which reverses the digits of the key and returns that value; for example, Reverse(7823) $=3287$.

The integer key values listed below are to be inserted, in the order given. Show the home slot (the slot to which the key hashes, before any probing), the probe sequence (if any) for each key, and the final contents of the hash table after the following key values have been inserted in the given order:

| Key <br> Value | Home <br> Slot | Probe Sequence |
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| Contents |  |  |  |  |  |  |  |  |  |  |  |

2. [10 points] The hash functions we have examined tend to use bitwise operations instead of arithmetic operations, because bitwise operations execute more quickly. You may have noticed that bitwise XOR is used frequently. Is there any reason to prefer XOR over bitwise AND or bitwise OR? Explain.
3. A hash table is being implemented, using double hashing to resolve collisions. Haskell Hoo proposes to make this more efficient by adopting the following scheme. We already have our primary hash function, H() , to hash the key value. Rather than use a second hash function, which would require another function evaluation, Haskell proposes that we just reuse the hash value from the primary function to compute the step length for collision resolution as follows:

$$
\text { step }=1+\mathrm{H}(\text { key }) \% 23
$$

a) [10 points] Why would Haskell have decided to add 1 in his proposal?
b) [10 points] The goal of double hashing is that keys that collide in the same slot will (hopefully) produce different probe sequences. Will Haskell's proposal be likely to achieve that goal? Explain.
4. Another hash table is being designed, and chaining, with singly-linked lists, will be used to resolve collisions. Haskell Hoo objects to the decision to use chaining, on the grounds that this could result in a more expensive search for some elements than if simple linear probing were used.
a) [8 points] Describe a scenario (perhaps unlikely) under which Haskell's claim could be correct. Note that some details of the proposed strategy have not been fully specified above, so that multiple options can be considered.
b) [8 points] Haskell says that his concern would be eliminated if the elements in the singly-linked lists were kept in sorted order. Is he correct? Explain.

