You will submit your solution to this assignment to the Curator System (as `HW01`). Your solution must be either a plain text file (e.g., NotePad) or a typed MS Word document; submissions in other formats will not be graded.

Partial credit will only be given if you show relevant work.

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1. [25 points] Design an algorithm to count and return the number of nodes in a binary tree that have two children. Express your solution as a pair of Java functions (not BST member functions), which would be implemented in the same package as the BST generic specified in Minor Project 2:

```
public int numFullNodes( BST<T> Tree ) {
   . . .
}
```

(Of course, the function shown above should have a recursive helper function.)

2. [25 points] Design an algorithm to count and return the greatest lower bound (GLB) of a data object `X`. Your solution will assume that the following public method has been added to the interface for the BST given in Minor Project 2:

```
// Pre:      X is a valid object of type T
//
// Returns:  reference to the unique object Y in the BST such that
//              Y = max { Z in tree | X.compareTo(Z) >= 0 }
//           or NULL if no such element exists in the BST
//
public T GLB(T X) {

   return GLBHelper(X, root, . . .);
}
```

Complete the implementation of the following private helper function, which would also be added to the given BST interface:

```
private T GLBHelper(T X, BinaryNode sroot, . . .) {
   . . .
}
```

The "`. . .`" indicates you may use additional parameters if you find them useful or necessary. Your implementation should operate as efficiently as possible; that is, it should not examine any branch of the BST unless that branch could contain relevant data.

3. [25 points] Use Induction to prove the following fact: for every integer, $h \geq 0$, a full binary tree with height h can have at most $2^{h+1} - 1$ nodes. (You may not use any of the tree theorems from the notes.)

4. [25 points] Use the result proved in question 3 to prove that: for every integer, $N \geq 0$, a full binary tree with N nodes must have at least $\lceil \log(N+1) \rceil - 1$ levels.