CS 3114 Data Structures and Algorithms

You will submit your solution to this assignment to the Curator System (as HW02). Your solution must be either a plain text file (e.g., NotePad) or a typed MS Word document; submissions in other formats will not be graded.

Except as noted, credit will only be given if you show relevant work.

1. [25 points] Using the rules given in the course notes, perform an exact count complexity analysis, for the worst case, of the body of the following function.

```
void Mystery(int M[N][N], const int N) {
  for (int R = 0; R < N; R++) {
    for (int C = 1; C < N; C++) {
        if (M[R][C-1] < M[R][C])
            M[R][C-1] = M[R][C];
        else {
            M[R][C] = M[R][C-1];
            M[R][C-1] = 0;
        }
    }
}</pre>
```

State both a complexity function T(N) and the Θ -complexity of T(N).

2. [25 points] Consider the following function, where α is an unknown positive constant:

$$f(n) = n^{\alpha} + \log n$$

Use Theorem 8 from the course notes on asymptotics to prove each of the following facts:

- a) f(n) is $\Theta(n^{\alpha})$ if $\alpha > 0$
- b) f(n) is $\Theta(\log n)$ if $\alpha < 0$
- 3. [25 points] For each part, determine the simplest possible function g(n) such that the given function is $\Theta(g)$. No justification is necessary, but you might have to do some analysis using the theorems from the notes.
 - a) $a(n) = 14n^2 + 3n \log n$
 - b) $b(n) = 3n^2 \log n$
 - c) $c(n) = 3n\log^2 n + 3n^2\log n$
 - d) $d(n) = 10n^2 + 2^n$
 - e) $e(n) = \frac{1}{n}$
- 4. [25 points] Suppose that executing an algorithm on input of size N requires executing $T(N) = 3N \log N + 16N$ instructions. How long would it take to execute this algorithm on hardware capable of carrying out 2²⁵ instructions per second if $N = 2^{30}$? (Give your answer in hours, minutes and seconds, to the nearest second.)