

You will submit your solution to this assignment to the Curator System (as HW2). Your solution must be either a plain text file (e.g., NotePad) or a MS Word document; submissions in other formats will not be graded.

Except as noted, credit will only be given if you show relevant work.

1. [20 points] Using the rules given in the course notes, perform an exact count complexity analysis, for the worst case, of the body of the following function. (Take `list.length` to be N .)

```
int part(int[] list, int barrierIdx) {
    int barrier, maxIdx, temp;

    barrier = list[barrierIdx];           // 1
    maxIdx = list.length - 1;           // 2

    temp = list[barrierIdx];             // 3
    list[barrierIdx] = list[maxIdx];     // 4
    list[maxIdx] = temp;                 // 5

    barrierIdx = 0;                      // 6

    for (int i = 0; i < maxIdx; i++) {   // 7
        if ( list[i] < barrier ) {       // 8
            temp = list[barrierIdx];     // 9
            list[barrierIdx] = list[i];  //10
            list[i] = temp;              //11
            barrierIdx++;                 //12
        }
    }
    temp = list[maxIdx];                 //13
    list[maxIdx] = list[barrierIdx];     //14
    list[barrierIdx] = temp;             //15

    return barrierIdx;                   //16
}
```

2. [40 points] For each part, determine the simplest possible function $g(n)$ such that the given function is $\Theta(g)$. No justification is necessary.

a) $a(n) = 3 + 14n + 47n^2$

b) $b(n) = 14n^2 + 3n \log n$

Hint: the last three take a little analysis.

c) $c(n) = n^{0.9} + \log n$

d) $d(n) = 3n^2 \log n + n^3$

e) $e(n) = 3n \log^2 n + 3n^2 \log n$

3. [20 points] An equivalent definition of Θ is:

Suppose that $f(n)$ and $g(n)$ are non-negative functions of N .

Then $f(n)$ is $\Theta(g(n))$ if there exist positive constants C_1 , C_2 and N such that, for all $n \geq N$, $C_1g(n) \leq f(n) \leq C_2g(n)$.

Use the alternate definition given above to prove the following statement:

Suppose that $f(n)$, $g(n)$, $r(n)$ and $s(n)$ are non-negative functions of N , such that $f(n)$ is $\Theta(r(n))$ and $g(n)$ is $\Theta(s(n))$.

Then the function $f(n) + g(n)$ is $\Theta(r(n) + s(n))$.

4. [20 points] Suppose that executing an algorithm on input of size N requires executing $T(N) = N \log N + 16N$ instructions. How long would it take to execute this algorithm on hardware capable of carrying out 2^{24} instructions per second if $N = 2^{30}$? (Give your answer in hours, minutes and seconds, to the nearest second.)