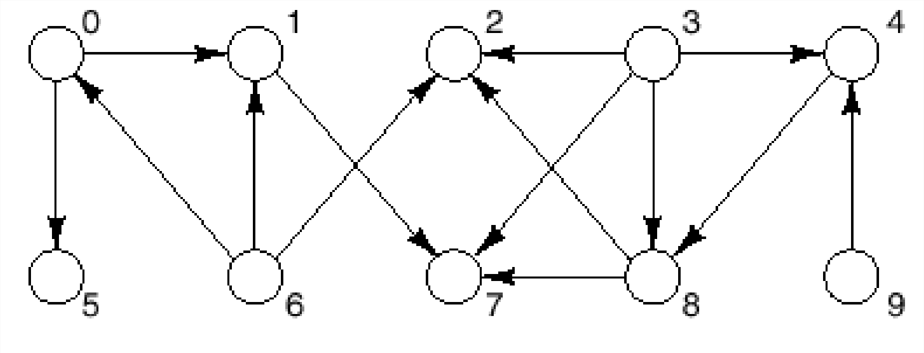
Computing a Topological Ordering



A topological ordering of the vertices of G may be obtained by performing a generalized depth-first traversal.

We build a list L of vertices of G.

Begin with vertex 0.

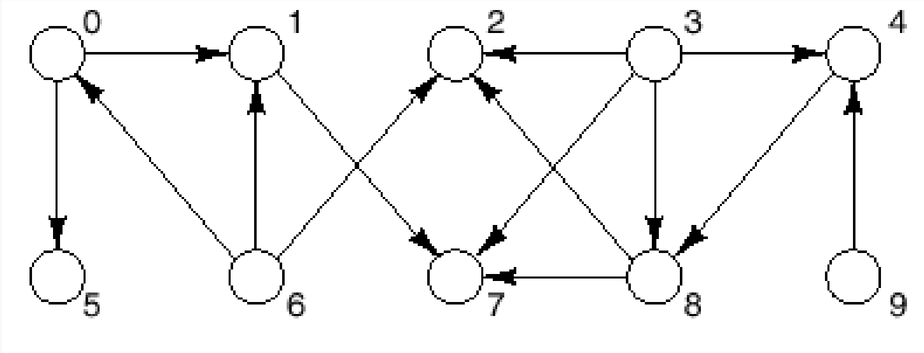
Do a depth-first traversal, marking each visited vertex and adding a vertex to L only if it has no unmarked successors.

When the traversal adds its starting vertex to L, pick the first unmarked vertex as a new starting point and perform another depth-first traversal.

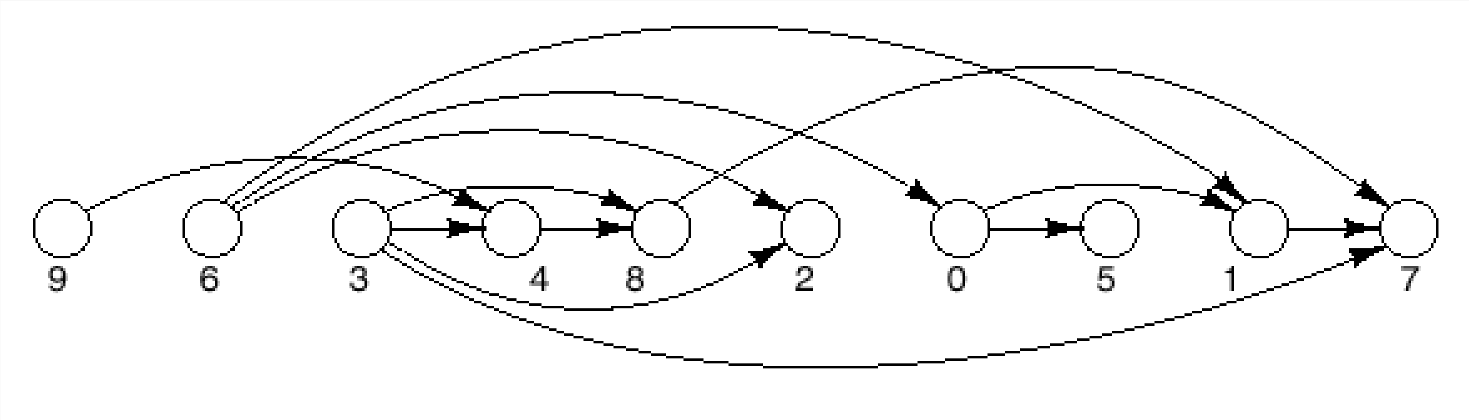
Stop when all vertices have been marked.

Topological Ordering

Suppose that G is a directed graph which contains no directed cycles:



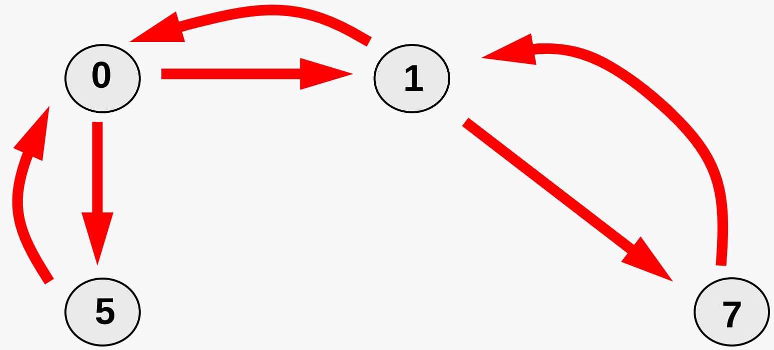
Then, a topological ordering of the vertices in G is a sequential listing of the vertices such that for any pair of vertices, v and w in G, if (v,w) is an edge in G then v precedes w in the sequential listing.



Depth-First Traversal Trace

Initially we probe from 0 to 1 to 7, which has no successors.

L: 7



Next the recursion backs out to 1, which has no unmarked successors.

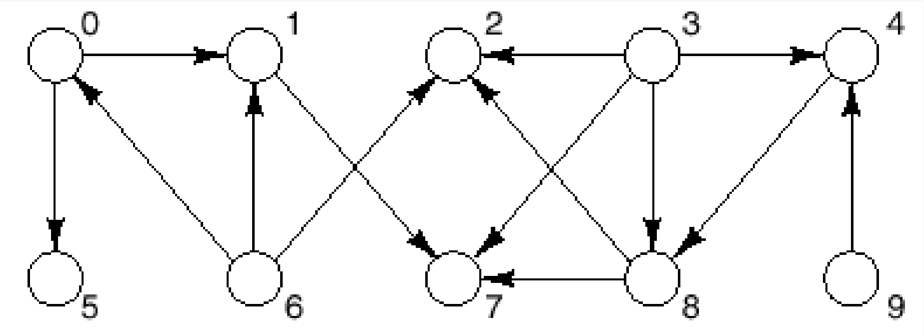
L: 1 7

Next the recursion backs out to 0, and probes to 5, which has no successors.

L: 5 1 7

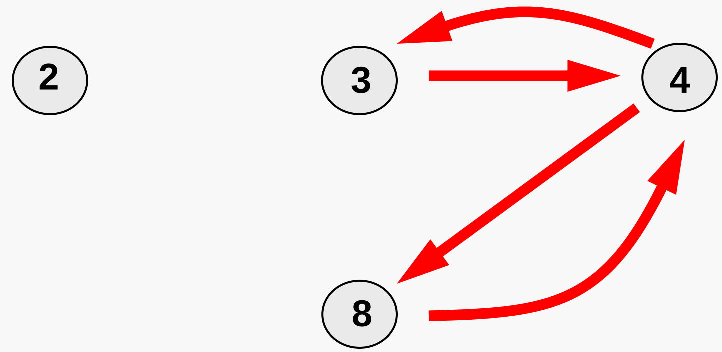
Next the recursion backs out to 0 again, which now has no unmarked successors.

L: 0 5 1 7



Depth-First Traversal Trace

Now we pick the first unmarked vertex, 2, and continue the process. 2 has no successors.



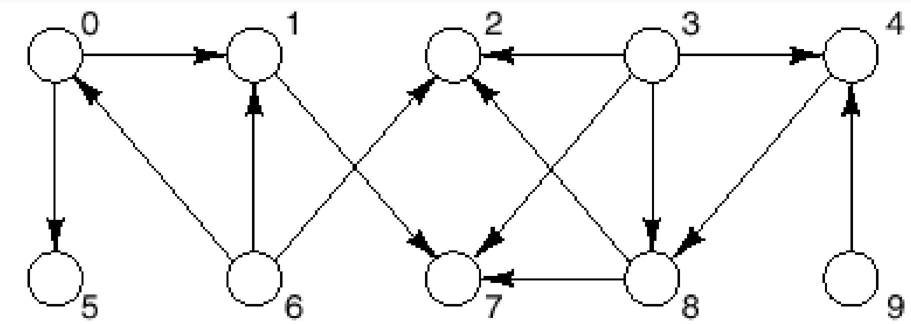
L: 2 0 5 1 7

Next start with vertex 3, and probe to 4 and then to 8, which has no unmarked successors.

L: 8 2 0 5 1 7

The recursion backs out, adding vertices 4 and 3.

L: 3 4 8 2 0 5 1 7



Depth-First Traversal Trace

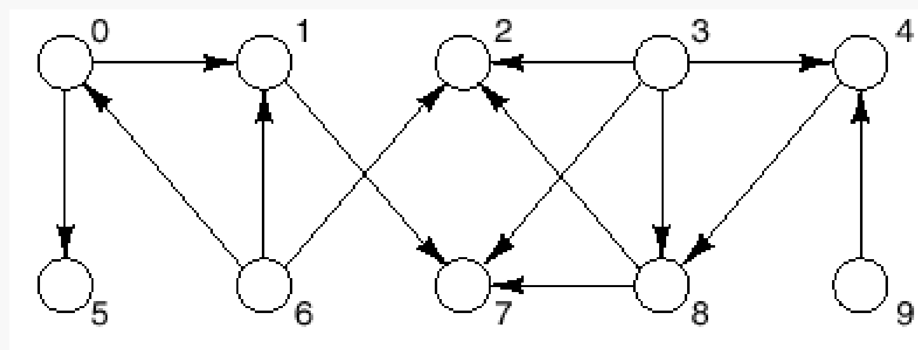
Next we pick the unmarked vertex, 6, which has no unmarked successors.

L: 6 3 4 8 2 0 5 1 7

Next we pick the unmarked vertex, 9, which has no unmarked successors.

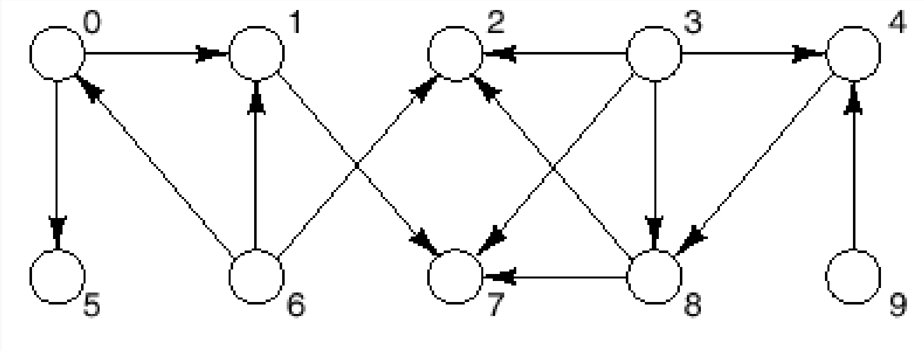
L: 9 6 3 4 8 2 0 5 1 7

At this point, all the vertices have been marked and the algorithm terminates.

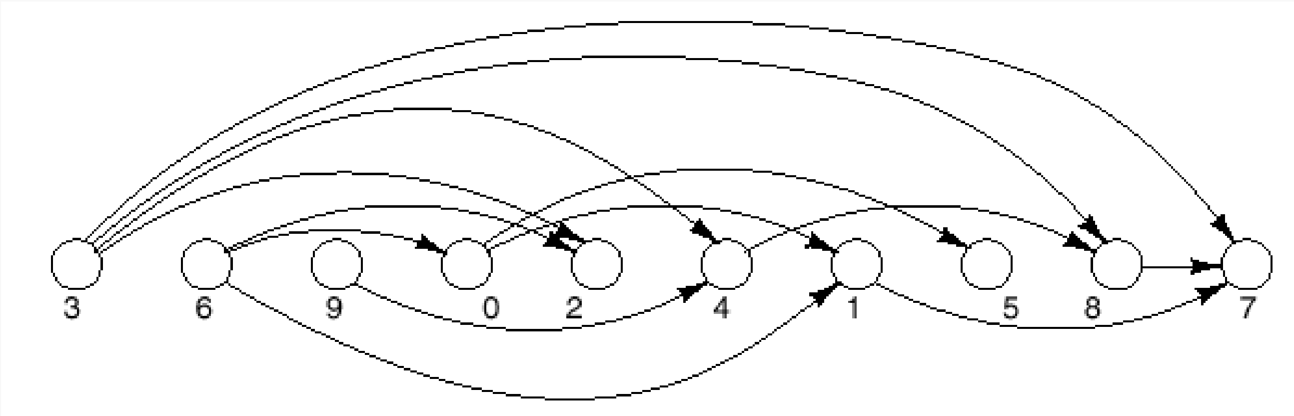


Multiple Topological Orderings

There is usually more than one topological ordering for a graph. Choosing a different rule for picking the starting vertices may yield a different ordering.



Also, a generalized breadth-first traversal can be used instead. For the graph above, a breadth-first traversal yields the ordering:



Applications of Topological Orderings

Applications of topological orderings are relatively common…

- prerequisite relationships among courses
- glossary of technical terms whose definitions involve dependencies
- organization of topics in a book or a course