You will submit your solution to this assignment to the Curator System (as `HW04`). Your solution must be either a plain text file (e.g., NotePad++) or a typed MS Word document; submissions in other formats will not be graded.

Credit will only be given if you show relevant work.

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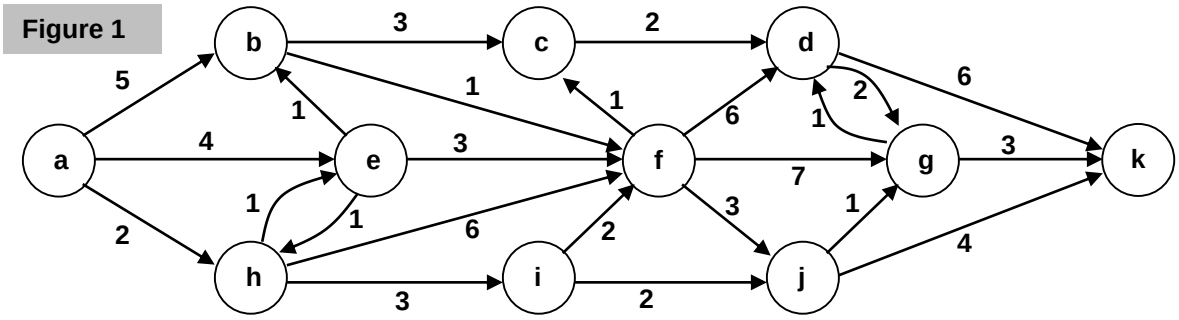
1. **[25 points]** Apply Dijkstra's SSAD algorithm to find the shortest distance from vertex a to every other vertex in the graph shown in Figure 1 below. For uniformity, when choosing which node to visit next, take them in increasing alphabetic order. You must show supporting work in the form of a table; see the course website for an acceptable format. You do not need to list the paths in your answer, just the minimum distances.

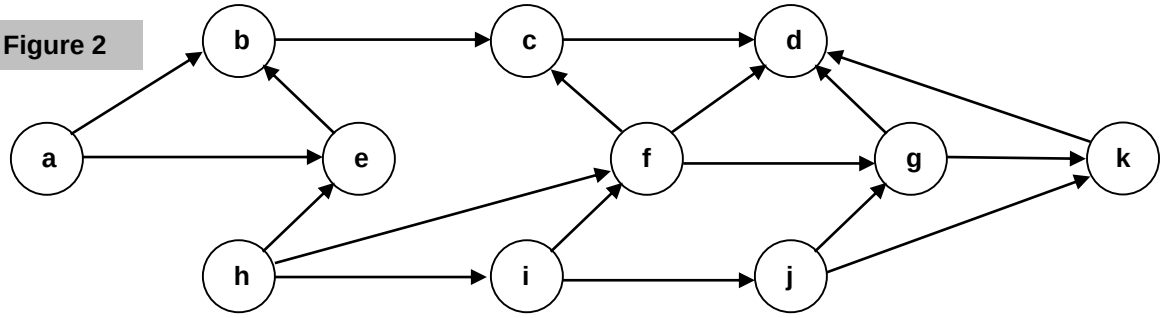
Note: the example in the course notes shows an undirected graph, but the algorithm applies to directed graphs as well, and in the obvious manner.

Figure 1



2. **[25 points]** Using a depth-first traversal, find a topological ordering of the nodes in the graph shown in Figure 2 below. For uniformity, when choosing which node to visit next, take them in increasing alphabetic order. You must show supporting work; see the course website for an acceptable format.

Figure 2



3. Suppose you have a list of values. The first N are in sorted order, but the last of those is followed by M more values which are in random order. Your problem is that you want to put the entire list, all $N + M$ elements, into sorted order, and you want to achieve that with a total cost that's $O(N)$, not including the cost of putting the first N elements into sorted order. You may use any of the sorting algorithms covered in class, and you may use two of them in combination if you like. For each of the following questions, describe your solution in words. Do not write code. Your justifications should at least resemble formal proofs.

a) **[10 points]** Explain how you could achieve your goal if $M = 1$ (i.e., there is one element in the unsorted part). Justify your claim that the entire process is $O(N)$.

b) **[10 points]** Explain how you could achieve your goal if $M = \log N$. Justify your claim that the entire process is $O(N)$.

c) **[10 points]** Explain how you could achieve your goal if $M = \sqrt{N}$. Justify your claim that the entire process is $O(N)$.

4. Suppose you are given a list S of N different, nonnegative integers. Explain whether each of the following search problems could be solved more efficiently if the elements in S were sorted in ascending order. Do not consider the cost of sorting the values in S as part of the analysis.

a) **[10 points]** Find a set of three values in S such that they fit in the smallest possible interval on the real line.

b) **[10 points]** Given a specific integer M between 0 and 99, inclusive, find all the values in S that are congruent to M modulo 100.