You will submit your solution to this assignment to the Curator System (as `HW2`). Your solution must be either a plain text file (e.g., NotePad) or a MS Word document; submissions in other formats will not be graded.

Except as noted, credit will only be given if you show relevant work.

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1. [20 points] Using the rules given in the course notes, perform an exact count complexity analysis, for the worst case, of the body of the following function. (Take the cost of `list.length` to be 2.)

```
int part(int[] list, int barrierIdx) {

   int barrier, maxIdx, temp;

   barrier = list[barrierIdx];              // 1
   maxIdx  = list.length - 1;               // 2

   temp = list[barrierIdx];                 // 3
   list[barrierIdx] = list[maxIdx];         // 4
   list[maxIdx] = temp;                     // 5

   barrierIdx = 0;                          // 6

   for (int i = 0; i < maxIdx; i++) {       // 7
      if ( list[i] < barrier ) {            // 8
         temp = list[barrierIdx];           // 9
         list[barrierIdx] = list[i];        //10
         list[i] = temp;                    //11
         barrierIdx++;                      //12
      }
   }
   temp = list[maxIdx];                     //13
   list[maxIdx] = list[barrierIdx];         //14
   list[barrierIdx] = temp;                 //15

   return barrierIdx;                       //16
}
```

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2. [35 points] For each part, determine the simplest possible function g(n) such that the given function is $\Theta(g)$. No justification is necessary.

   a) $a(n) = 3 + 14n + 47n^2$

   b) $b(n) = 14n^2 + 3n \log n$

   Hint: the last three take a little analysis.

   c) $c(n) = n^{0.9} + \log n$

   d) $d(n) = 3n^2 \log n + n^3$

   e) $e(n) = \dfrac{1+n}{5n}$

3. Suppose that $f(n)$ and $g(n)$ are non-negative functions and that $f(n)$ is $O(n^2)$ and $g(n)$ is $O(n)$.

   a) [10 points] Define a function $S(n)$ by $S(n) = f(n) + g(n)$. What's the "smallest" function $M(n)$ such that $S(n)$ is guaranteed to be $O(M(n))$? Justify your conclusion with a proof! (If you want to make your argument with limits, you may assume that any related limits do exist.)

   b) [10 points] Define a function $P(n)$ by $P(n) = f(n) \cdot g(n)$. What's the "smallest" function $Z(n)$ such that $S(n)$ is guaranteed to be $O(Z(n))$? Justify your conclusion with a proof! (If you want to make your argument with limits, you may assume that any related limits do exist.)

   c) [5 points] What can you conclude about the relationship between $f(n)$ and $g(n)$? For example, can you conclude that $f(n)$ is $O(g(n))$? or that $g(n)$ is $O(f(n))$? or some other relationship? Explain your conclusion.

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4. [20 points] Suppose that executing an algorithm on input of size N requires executing $T(N) = N \log N + 16N$ instructions. How long would it take to execute this algorithm on hardware capable of carrying out 2^{24} instructions per second if $N = 2^{28}$? (Give your answer in hours, minutes and seconds, to the nearest second.)