Instructions:

- Print your name in the space provided below.
- This examination is closed book and closed notes. No calculators or other computing devices may be used.
- Answer each question in the space provided. If you need to continue an answer onto the back of a page, clearly indicate that and label the continuation with the question number.
- If you want partial credit, justify your answers, even when justification is not explicitly required.
- There are 7 questions, priced as marked. The maximum score is 100.
- When you have completed the test, sign the pledge at the bottom of this page and turn in the test.
- **Note that failure to return this test, or to discuss its content with a student who has not taken it, is a violation of the Honor Code.**

Do not start the test until instructed to do so!

Name Solution
printed

Pledge: On my honor, I have neither given nor received unauthorized aid on this examination.

signed

1. [20 points] Indicate whether each of the following statements is true or false:

a) $f(n) = 5n^2 + 3n + 2$ is $O(n^2)$ [✓ TRUE] FALSE

b) $f(n) = 5n^2 + 3n + 2$ is $\Omega(n^2)$ [✓ TRUE] FALSE

c) $g(n) = 3n^2 + 100\ n \log n$ is $\Theta(n^2)$ [✓ TRUE] FALSE

d) $g(n) = 3n + 100 \log n$ is $\Theta(n)$ [✓ TRUE] FALSE

e) $g(n) = 3n + 100 \log n$ is $O(\log n)$ TRUE [✓ FALSE]

2. [12 points] Suppose that T is a full binary tree. Remember the Full Binary Tree Theorem?

FBT A full binary tree with I internal nodes has I + 1 leaf nodes.

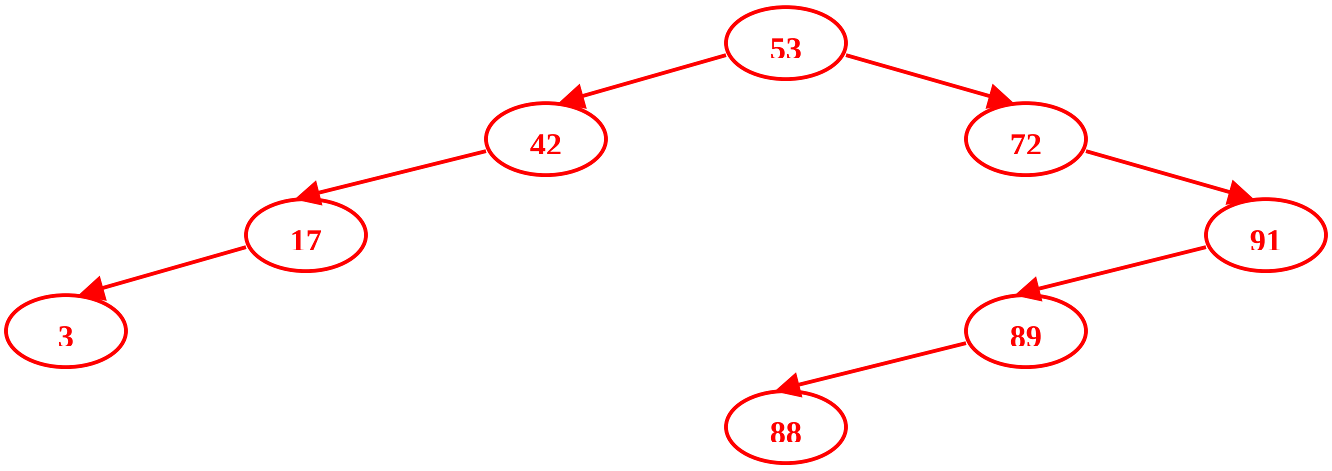
a) If T has 100 leaf nodes then the number of internal nodes in T is ____**99**_____.

b) If T has a total of 1001 nodes, then ______**1000**___ of them must be child nodes. **(All but the root.)**

c) Is it possible that the total number of nodes in T is even? **No, the total must be 2I + 1, which is odd.**

3. [10 points] Draw the BST that results from inserting the following data values in the order given:

53 72 91 42 17 89 3 88



4. You must keep track of some data. Your options are:

   I) a binary search tree of records (assume it is optimally balanced)
   II) a linked-list of records stored in order of insertion

   a) [12 points] Suppose that you must first build a data structure holding E given elements, and then you must perform S searches on that data structure. For each option above, use the average case big-Θ **time** complexity results of each data structure to determine the costs associated with that data structure in this situation. The cost values should be numbers, not expressed in terms of E and S.

Note that space cost is not a consideration.

Option	Cost of construction from E elements	Cost of S searches
BST	**E log E**	**S log E**
unsorted linked list	**E**	**SE**

   b) [6 points] Based on your analysis above, which of the data structures would be the <u>better</u> choice in the given situation, provided that the number of elements to store was 2^{10} and the number of searches to perform was 2^{20}? **Explain.**

Now, $E = 2^{10}$ and $S = 2^{20}$.

For the BST: cost
$$\begin{aligned} &= E \log E + S \log E = 2^{10} \log 2^{10} + 2^{20} \log 2^{10} \\ &= (2^{10} + 2^{20}) \log 2^{10} \\ &= 10\,(2^{10} + 2^{20}) \\ &\approx 10\ 2^{20} \end{aligned}$$

For the ULL: cost
$$\begin{aligned} &= E + SE = 2^{10} + 2^{20}\ 2^{10} \\ &= 2^{10} + 2^{30} \\ &\approx 2^{30} \end{aligned}$$

Clearly the total cost is much lower (by about a factor of about 2^7) with the BST.

5. [15 points] Consider the `BST` and `NodeT` template interfaces given at the bottom of this page. Write the body of the private member function `CountHelper()`, which returns the number of times its parameter occurs in the BST. Your implementation must conform to the interface given below, and must not make any unnecessary visits to tree nodes. You may assume that the implementation of `Data` provides overloads for any relational operators you wish to use.

```
template <class Data> int BST<Data>::Count(const Data& toFind) {

   return CountHelper(toFind, Root);
}

template <class data>
int BST<Data>::CountHelper(const Data& toFind, NodeT<Data>* sRoot) {

   if (sRoot == NULL) return 0;           // null test

   Data currData = sRoot->Element;

   if (currData == toFind)
      return ( 1 + CountHelper(toFind, sRoot->Right) );

   if (currData < toFind)
      return ( CountHelper(toFind, sRoot->Right) );

   // currData > toFind
   return ( CountHelper(toFind, sRoot->Left) );









}
```

```
template <class Data> class NodeT {
public:
   Data          Element;
   NodeT<Data>* Left;
   NodeT<Data>* Right;

   // irrelevant members omitted
   Data getData() const;
   NodeT<Data>* getLeft() const;
   NodeT<Data>* getRight() const;
};
```

```
template <class Data> class BST {
private:
   NodeT<Data>* Root;

   // irrelevant members omitted
public:
   // irrelevant members omitted
   int Count(const Data& D);
};
```

6. A list of N records, each holding 16 bytes of data, must be stored, on a system where each pointer requires 4 bytes of storage. We consider two alternatives: a singly linked list of N nodes and an array of fixed dimension D, where $N \leq D$.

   [5 points] How much space would the singly linked list require?

   Counting the head pointer: 4 + N(16 + 4) = 4 + 20N

   [5 points] How much space would the array require?

   Counting the array pointer: 4 + D*16

   [5 points] Considering only total storage cost as important, how large must N be in order for the array to be the better choice? [Hint: since you don't know D, the answer must involve D.]

   Solve: 4 + 20N > 4 + 16D to get N > (4/5) D so the array must be 80% full.

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7. [10 points] Determine the big-O operation count for the execution of the following code fragment (in terms of N, of course). An exact operation count is not required.

```
int Sum = 0;
for (int i = 1; i <= N; i = 2*i) {   // log N passes

   for (int j = 1; j <= N; j++) {    // N passes

      Sum = Sum + i*j;               // constant # of operations

   }
}
```

   So the rough total cost would be N log N