

You will submit your solution to this assignment to the Curator System (as HW1). Your solution must be either a plain text file (e.g., NotePad) or a MS Word document; submissions in other formats will not be graded.

Partial credit will only be given if you show relevant work.

1. [20 points] Using any relevant theorems from the notes, conjecture a simple function  $g$  such that  $f$  is  $\Theta(g)$ , and prove that your conjecture is correct if:

$$f(n) = n^2 \log n + n^3 + 1000$$

2. [20 points] Let  $\alpha$  be an arbitrary positive constant, and define two functions:

$$f(n) = \log n \quad \text{and} \quad g(n) = n^\alpha$$

Using any theorems from the notes, prove that  $f$  is  $O(g)$  but  $f$  is not  $\Theta(g)$ .

3. [20 points] Suppose that  $f$  and  $g$  are non-negative functions such that  $f$  is  $\Theta(g)$ . Is it necessarily true that:

$$2^{f(n)} \text{ is } \Theta\left(2^{g(n)}\right)$$

If so, prove it. (You may assume that the limit referred to in Theorem 8 exists.) If no, give a specific counter-example and show that it is a counter-example.

4. [20 pts] Assume a system uses a hard drive with the following physical characteristics:

<b>total capacity</b>	128 GB
<b># of platters</b>	8
<b># of tracks per surface</b>	16384
<b># of sectors per track</b>	2048
<b>cluster size</b>	4 KB
<b>spindle speed</b>	10000 RPM
<b>head start time</b>	1 ms
<b>track to track seek time</b>	0.01 ms

In answering the following questions, express all final time values to the nearest hundredth of a millisecond (8.33 ms).

- What is the average random head seek time for this drive?
- What is the average rotational latency for this drive?
- What is the average total time required to read one randomly-chosen sector from this drive?
- What is the average total time required to read a file of 10 MB from this drive if the clusters are randomly scattered on the drive?

5. [20 points] Consider solving a problem using an algorithm whose complexity is  $\Theta(N^2)$ . Estimate the running time of the algorithm if:
- $N = 2^{12}$  (4096) and the hardware is capable of executing  $2^{24}$  instructions per second.
  - $N = 2^{16}$  (65536) and the hardware is capable of executing  $2^{24}$  instructions per second.

Express your answers in days, hours, minutes and seconds. (Not in total seconds unless the time is shorter than 1 minute.)