Markov Models

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Last Time

- Probabilistic graphical models
- Bayesian networks
- Naive Bayes and Logistic Regression as Bayes nets
- Inference in Bayes nets



John Trumbull's *Declaration of Independence* in 1776

CAAAAA

AAAAA

ADAMAN

2.4



Independence

independent & identically distributed (i.i.d.)

amount of dependence

cheap, easy, embarrassingly parallel

full joint distributions

super expensive





Outline

- Time series
- Markov models
- Variable elimination in Markov models
- Forward message-passing inference



more Historical Weather Data >

Read Story >

Time Series

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| + Indicator + | Comparison 1d | 5d | 1m 3r | n 6m | YTD | 1y | 2y | 5y 10y | Max | ⊞ ~ | ~ | Linear | C | Go To Symbo | \rightarrow | E | \mathbb{Z} |
|--|---------------|-----|--------|------|-----|----|----|--|-----|-----|---|--------|---|-------------|---------------|---|--------------|
| Open 2079.47 Close 2079.42 Low 2079.42 | ^GSPC 2079.42 | | | | | | | | | | | | | | | | 2 |
| High 2079.59 Vol 643.49K % Chg 0.65% | | | | | | | | | | | | | | | \mathcal{N} | | 2 |
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| QEP Resources tops 3Q profit forecasts Open 2,066.48 Low 2,063.11 | Recent Articles | Prev Close | 2,065.89 | High | 2,090.35 |
|---|--|------------|----------|------|----------|
| | QEP Resources tops 3Q profit forecasts | Open | 2,066.48 | Low | 2,063.11 |



- Goals:
 - Prediction

• Filtering, smoothing



Time Series

Markov Models

Markov assumption: the past is independent of the future given the present

 $p(x_i, x_k | x_i) = p(x_i | x_i) p(x_k | x_i)$ i < j < k

T-1 $p(x_1,...,x_T) = p(x_1) \prod p(x_{t+1}|x_t)$ t=1

> usually parameterized with function independent of *t*

 $p(x_1, x_2, x_3, x_4) = p(x_1)p(x_2|x_1)p(x_3|x_2)p(x_4|x_3)$ X_1, X_2, X_3 $p(x_2) = \alpha_2(x_2) = \sum p(x_1)p(x_2|x_1)$ X_1 $p(x_4) = \sum \alpha_2(x_2)p(x_3|x_2)p(x_4|x_3)$ X_2, X_3 $p(x_3) = \alpha_3(x_3) = \sum \alpha_2(x_2)p(x_3|x_2)$ *X*₂

Variable Elimination



 $p(x_4) = \sum p(x_1)p(x_2|x_1)p(x_3|x_2)p(x_4|x_3)$

 $p(x_4) = \sum \alpha_3(x_3)p(x_4|x_3)$



Forward Message Passing

- $p(X) = p(x_1) \prod_{t=1}^{T-1} p(x_{t+1}|x_t)$
- for **t** from 1 to (T-1): $p(x_{t+1}) = \sum_{x_t} p(x_t) p(x_{t+1} | x_t)$

What about if we observe evidence?



 $p(x_4|x_1=1)$

- vidence?
 - $p(x_4|x_1 = 1, x_8 = 0)$

Summary

- Time series
- Markov models
- Variable elimination in Markov models
- Forward message-passing inference