**An Example of the Need for Numerical Methods**

Suppose $p(t) =$ population at time $t$. A sequence of increasingly complicated Initial Value Problems (ordinary differential equations) might represent the situation being studied. We know the population $p_0$ at time 0, and we hypothesize a particular formula for the population growth rate $p'(t)$.

- Model 1. Assume birth rate is constant. Can solve for $p(t)$ using Calculus.

$$\left.\begin{array}{rcl} p'(t) &=& c \\ p(0) &=& p_0 \end{array}\right\} \implies p(t) = ct + p_0$$

- Model 2. Assume birth rate is proportional to current population. Can solve for $p(t)$ using Calculus.

$$\left.\begin{array}{rcl} p'(t) &=& \lambda p(t) \\ p(0) &=& p_0 \end{array}\right\} \implies p(t) = p_0 e^{\lambda t}$$

- Model 3. Add a term to population growth rate corresponding to a constant rate of immigration from outside the system. Can still solve for $p(t)$ using Calculus.

$$\left.\begin{array}{rcl} p'(t) &=& \lambda p(t) + \nu \\ p(0) &=& p_0 \end{array}\right\} \implies p(t) = p_0 e^{\lambda t} + \frac{\nu}{\lambda}(e^{\lambda t} - 1)$$

- Model 4. Now suppose the rate of immigration depends somehow on the current population. As model becomes more complex ...

$$\left.\begin{array}{rcl} p'(t) &=& \lambda p(t) + \nu \ln(p(t)) \\ p(0) &=& p_0 \end{array}\right\} \implies p(t) = ? \left(\begin{array}{l} \text{need numerical method} \\ \text{to ``solve'' for } p(t). \end{array}\right)$$

Now suppose the problem is to solve for the birth rate at time $t = 1$, given $p_0 = 10$, $p(1) = 13$, and $\nu = 1$.

- With model 1: $13 = c + 10 \implies c = 3$
- With model 2: $13 = 10e^{\lambda} \implies \lambda = \ln(\frac{13}{10}) = 0.26$
- With model 3: $13 = 10e^{\lambda} + \frac{1}{\lambda}(e^{\lambda} - 1) \implies$ ? (need numerical method to approximate $\lambda$).