

A *positional* or *place-value* notation is a numeral system in which each position is related to the next by a constant multiplier, called the *base* or *radix* of that numeral system.

The value of each digit position is the value of its digit, multiplied by a power of the base; the power is determined by the digit's position.

The value of a positional number is the total of the values of its positions.

So, in positional base-10 notation:

$$73901 = 7 \times 10^4 + 3 \times 10^3 + 9 \times 10^2 + 0 \times 10^1 + 1 \times 10^0$$

And, in positional base-2 notation:

$$10010000010101101 = 1 \times 2^{16} + 1 \times 2^{13} + 1 \times 2^7 + 1 \times 2^5 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^0$$

Why is the second example a cheat?

Do not confuse the representation with the number!

Each of the following examples is a representation of the same number:

255_{10}

11111111_2

FF_{16}

2010_5

377_8

3333_4

100110_3

Do not make the mistake of thinking that there is such a thing as "a base-10 number" or "a base-16 number".

There is a unique base-10 representation of every integer and there is a unique base-16 representation of every integer.

Converting from base-10 to base-2

Given a base-10 representation of an integer value, the base-2 representation can be calculated by successive divisions by 2:

	Remainder	
73901		
36950	1	} 10010000010101101 ₂
18475	0	
9237	1	
4618	1	
2309	0	
1154	1	
577	0	
288	1	
144	0	
72	0	
36	0	
18	0	
9	0	
4	1	
2	0	
1	0	
0	1	

Given a base-2 representation of an integer value, the base-10 representation can be calculated by simply expanding the positional representation:

$$\begin{aligned} 10010000010101101_2 &= 1 \times 2^{16} + 1 \times 2^{13} + 1 \times 2^7 + 1 \times 2^5 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^0 \\ &= 65536 + 8192 + 127 + 32 + 8 + 4 + 1 \\ &= 73901 \end{aligned}$$

Are analagous... given a base-10 representation of an integer value, the base-16 representation can be calculated by successive divisions by 16:

	Remainder	
73901		
4618	13 --> D	} 120AD ₁₆
288	10 --> A	
18	0	
1	2	
0	1	

The choice of base determines the set of numerals that will be used.

base-16 (hexadecimal or simply hex)

numerals: 0 1 ... 9 A B C D E F

Given a base-2 representation of an integer value, the base-16 representation can be calculated by simply converting the nybbles:

```
1 0010 0000 1010 1101
1   2   0   A   D   : hex
```

The same basic "trick" works whenever the target base is a power of the source base:

```
10 010 000 010 101 101
2   2   0   2   5   5   : octal
```

Important Bases in Computing

base-2	binary	0 1
base-8	octal	0 1 2 3 4 5 6 7
base-10	decimal	0 1 2 3 4 5 6 7 8 9
base-16	hex	0 1 2 3 4 5 6 7 8 9 A B C D E F