You may work in pairs or purely individually for this assignment. Prepare your answers to the following questions in a plain ASCII text file or MS Word document. Submit your file to the Curator system by the posted deadline for this assignment. No late submissions will be accepted. If you work in pairs, list the names and email PIDs of both members at the beginning of the file, and submit your solution under only one PID. No other formats will be graded.

For this assignment, you may (and are encouraged to) work in pairs; if you do so, you must also write your solutions in such a way that it is clear how each member contributed to deriving the solution.

You will submit your answers to the Curator System (www.cs.vt.edu/curator) under the heading `OOC05`.

For each of the following questions, you are asked to derive a recurrence relation. You must explain clearly why your recurrence relation is correct, but you are not required to solve the recurrence.

1. [25 points] Find a recurrence relation for the number of ways to arrange a sequence of flags on an *N*-foot flagpole, if you have four kinds of flags: blue flags and green flags that are 3 feet tall, yellow flags that are 2 feet tall, and red flags that are 1 foot tall.

Let F_n be the number of ways to arrange the given kinds of flags on a pole that is n feet tall.

Then $F_1 = 1$ since only a red flag will fit.
And $F_2 = 2$ since we can have (from bottom to top): Y RR
And $F_3 = 5$ since we can have any of the following: B G YR RY RRR

So, what's the connection? Suppose we have filled a pole of height n. The top flag must either be of height 3 (B or G), height 2 (Y) or height 1 (R). So we have three cases:

- **this is an extension of a solution for a pole of height n – 3 (by adding a B or G)**
- **this is an extension of a solution for a pole of height n – 2 (by adding a Y)**
- **this is an extension of a solution for a pole of height n – 1 (by adding a R)**

So we have the following recurrence relation:

$$F_1 = 1,\ F_2 = 2,\ F_3 = 5$$
$$F_n = F_{n-1} + F_{n-2} + 2F_{n-3} \text{ for } n > 3$$

2. [25 points] Code sequences are formed using characters from the set {a, b, c, d}. A code sequence is *legitimate* if and only if it contains an even number of b's. How many legitimate code sequences of length *n* are there?

QTP: if l_n is the number of legitimate code sequences of length *n*, what is the number of illegitimate code sequences of length *n*?

I covered a related example, from the previous ICE in class.

3. [25 points] Earlier, we discussed the problem of counting the number of ways to choose *K* items from among a collection of *N* distinct items, and we said that the answer was given by the binomial coefficient:

$$\binom{N}{K}$$

The following identity for binomial coefficients is easy to prove algebraically:

$$\binom{N}{K} = \binom{N-1}{K} + \binom{N-1}{K-1}$$

Explain why this identity makes sense as a recurrence relation.

Suppose we have a set S of N objects; pick one, say x, and remove it, leaving a set T of N - 1 objects.

Now, each subset of the original set S either contains x or does not contain x. Subsets of S that do not contain x are really subsets of T, and the number of subsets of T that contain K objects is just:

$$\binom{N-1}{K}$$

On the other hand, a subset of S that does contain x is just a subset of K-1 things from T with x added in. The number of subsets of K-1 objects from T is just:

$$\binom{N-1}{K-1}$$

Since this accounts for all the subsets of K objects from S, this explains the identity.

4. [25 points] Earlier, we considered the Towers of Hanoi. The Double Towers of Hanoi problem is identical, except that there are $2N$ disks of N different sizes (so, two disks of each size). It is acceptable to have a disk on top of a larger disk, or on top of a disk of the same size, but not on top of a disk of a smaller size. You are allowed to pick up one disk at a time, and transfer it to a different pole. Initially, all $2N$ disks are stacked on one of the three poles.

Find a recurrence relation for the number of moves required to transfer all the disks from the original pole to another of the three poles, under the restrictions given above.

I covered this one in class.