<u>You may work in pairs or purely individually for this assignment.</u> Prepare your answers to the following questions in a plain ASCII text file or MS Word document. Submit your file to the Curator system by the posted deadline for this assignment. No late submissions will be accepted. If you work in pairs, list the names and email PIDs of both members at the beginning of the file, and submit your solution under only one PID. No other formats will be graded.

For this assignment, you may (and are encouraged to) work in pairs; if you do so, you must also write your solutions in such a way that it is clear how each member contributed to deriving the solution.

You will submit your answers to the Curator System (www.cs.vt.edu/curator) under the heading `OOC04`.

Be clear about the logic of your analysis for each question. Show the steps you applied, and explain why you did so.

1. [20 points] Find the solution for the following Diophantine equation where x is positive and as small as possible:

$$7x + 15y = 1$$

2. [20 points] Find the solution for the following Diophantine equation where x is positive and as small as possible:

$$56x - 21y = 49$$

3. [20 points] When the accountants for Lose-a-Byte Computer, Inc, had finished preparing their annual budget proposal, they presented the final figures to the company CEO, Yoov Bin Jobbed (YB). "It looks like a good year ahead!", YB announced. "The amount of the budget, in dollars, just happens to be the smallest integer (other than 1) that's a perfect square, and a perfect cube, and a perfect fifth power!"

What was the amount of the proposed budget?

Hint: if X and k are positive integers and $X^{1/k}$ is an integer then X must be a perfect k-th power of some integer. You may use that fact in your analysis, but for full credit you must prove this fact in order to use it.

4. [20 points] A bag contains 100 pieces of U. S. currency, but only half-dollars, $5 bills and $10 bills. The total value of the currency is $100. How many of each kind of currency were in the bag?

5. [20 points] Consider the following function, where k must be a nonnegative integer:

$$f(k) = \frac{k^2 + k}{2}$$

The function can be used to generate a rather dull sequence of integers: 0, 1, 3, 6, 10, 15, 21, 28, 36, . . .

If we are given a positive integer N, say 8, we can take the remainder modulo N of each of the numbers in that sequence we get:

0, 1, 3, 6, 2, 7, 5, 4, 4, . . .

It turns out that if N is a power of 2 (like 8 in the example above), then there are NEVER any repeated values in the first N terms in the sequence we get from $f(k)$ mod N.

Apply the number theoretic ideas we discussed in class to explain why that fact is so. You don't need to present a full, formal proof (although that would be nice), but you need to give a convincing argument. (And, the fact that N is a power of 2 is vital, so any explanation that doesn't depend of that property of N would be incorrect.)