<u>You may work in pairs or purely individually for this assignment.</u> Prepare your answers to the following questions in a plain ASCII text file or MS Word document. Submit your file to the Curator system by the posted deadline for this assignment. No late submissions will be accepted. If you work in pairs, list the names and email PIDs of both members at the beginning of the file, and submit your solution under only one PID. No other formats will be graded.

For this assignment, you may (and are encouraged to) work in pairs; if you do so, you must also write your solutions in such a way that it is clear how each member contributed to deriving the solution.

You will submit your answers to the Curator System (www.cs.vt.edu/curator) under the heading `OOC03`.

Express your answers in terms of appropriate notation (factorials, combinations, permutations), but not simplify your answers to a numeric value. Also be clear about the logic of your analysis.

1. An urn contains 15 red balls, 10 blue balls, 12 green balls, and 13 yellow balls.

a) [12 points] In how many different ways can three balls be drawn from the urn, if they must be of the same color?

You can either have 3 red balls, or 3 blue ones, or 3 green ones, or 3 yellow ones, so the Addition Rule applies:

$$\binom{15}{3}+\binom{10}{3}+\binom{12}{3}+\binom{13}{3}$$

b) [4 points] If three balls are drawn randomly from the urn, what is the probability that they will be of the same color?

The probability is the number of outcomes that fit the event (from the previous part) divided by the total number of ways to have picked 3 balls:

$$\frac{\binom{15}{3}+\binom{10}{3}+\binom{12}{3}+\binom{13}{3}}{\binom{50}{3}}$$

c) [12 points] In how many different ways can four balls be drawn from the urn, if they must be of four different colors?

You need to choose one red ball, one blue ball, one green ball and one yellow ball, so the Multiplication Rule applies:

$$\binom{15}{1}\cdot\binom{10}{1}\cdot\binom{12}{1}\cdot\binom{13}{1}$$

d) [4 points] If four balls are drawn randomly from the urn, what is the probability they will be of four different colors?

The probability is the number of outcomes that fit the event (from the previous part) divided by the total number of ways to have picked 3 balls:

$$\frac{\binom{15}{1}\cdot\binom{10}{1}\cdot\binom{12}{1}\cdot\binom{13}{1}}{\binom{50}{4}}$$

e) [12 points] If six balls are drawn randomly from the urn, what is the probability the balls will not all be of the same color?

The probability the balls are NOT all of the same color would be 1 minus the probability that they ARE all of the same color (modified from a previous part):

$$1-\frac{\binom{15}{6}+\binom{10}{6}+\binom{12}{6}+\binom{13}{6}}{\binom{50}{6}}$$

f) [12 points] If six balls are drawn from the urn, and all of them are yellow, what is the probability that the next ball drawn will also be yellow?

There would now be 7 yellow balls in the urn, and 43 balls altogether, so the probability of drawing one of the yellow balls next would be:

$$\frac{\binom{7}{1}}{\binom{44}{1}}$$

2. Consider randomly selecting cards from a standard 52-card poker deck.

a) [15 points] In how many different ways can you construct a 5-card hand so that the hand contains 3 cards of one value and 2 cards of different values than all the other cards in the hand?

Following the discussion in the notes, we want to construct such a hand as follows:

- choose a value for the three-of-a-kind
- choose three cards of that value
- choose two values for the remaining cards
- choose one card of the higher value
- choose one card of the lower value

The number of ways to do this would be:

$$\binom{13}{1}\cdot\binom{4}{3}\cdot\binom{12}{2}\cdot\binom{4}{1}\cdot\binom{4}{1}$$

b) [15 points] In how many different ways can you construct a 7-card hand so that the hand contains 2 cards of one value, 2 cards of another value, and a 3 cards of a third value?

Here, our procedure should be:

- **choose two values for the pairs**
- **choose two cards of the higher value**
- **choose two cards of the lower value**
- **choose a value for the final three cards**
- **choose three cards of that value**

$$\binom{13}{2}\cdot\binom{4}{2}\cdot\binom{4}{2}\cdot\binom{11}{1}\cdot\binom{4}{3}$$

3. [14 points] The Evil Problem Solving Instructor summons you to his office. When you arrive, he shows you three boxes, each with a hole in the top through which you can insert your hand. He tells you that each box contains two valuable gems; one box contains two diamonds, one contains two emeralds, and one contains one diamond and one emerald. Of course, there is no way to see what is in any of the boxes.

You are asked to choose a box. After you have chosen a box, the Evil Problem Solving Instructor reaches into it and removes one gem; you immediately notice that it is a diamond. The Evil Problem Solving Instructor then tells you that you may have both gems from that box if you can correctly answer the question: what is the probability that the other gem in that box is also a diamond.

What should you say? Justify your conclusion very carefully.

(See the discussion of the problem with the cards with X's and O's in the notes. This is the same scenario.)