READ THIS NOW!

- Print your name in the space provided below.
- There are 5 problems, priced as marked. The maximum score is 100.
- The grading of each question will take into account whether you obtained a correct solution and how well you presented your analysis and justified your logic. In most cases, as much weight will be given to the presentation and explanation of your analysis as to whether the solution is fully correct. Legibility will be strongly considered in the grading. You may use scratch paper to work out your solution before finalizing it on the exam.
- Externalize! Whether it's a drawing, a table, an equation or something else, externalize! And make the externalization explicit in your answer! Label things for clarity!
- You may use the supplied extra paper for scratch work. Write your name on any scratch work sheets you use and turn those in with your exam.
- All final answers must be written on the test form itself.
- When you have finished, sign the pledge at the bottom of this page and turn in the test.
- This examination is closed book and closed notes, aside from the permitted one-page formula sheet. No calculators or other computing devices may be used. The use of any such device will be interpreted as an indication that you are finished with the test and your test form will be collected immediately.
- Until solutions are posted, you may not discuss this examination with any student who has not taken it.
- Failure to adhere to any of these restrictions is an Honor Code violation.

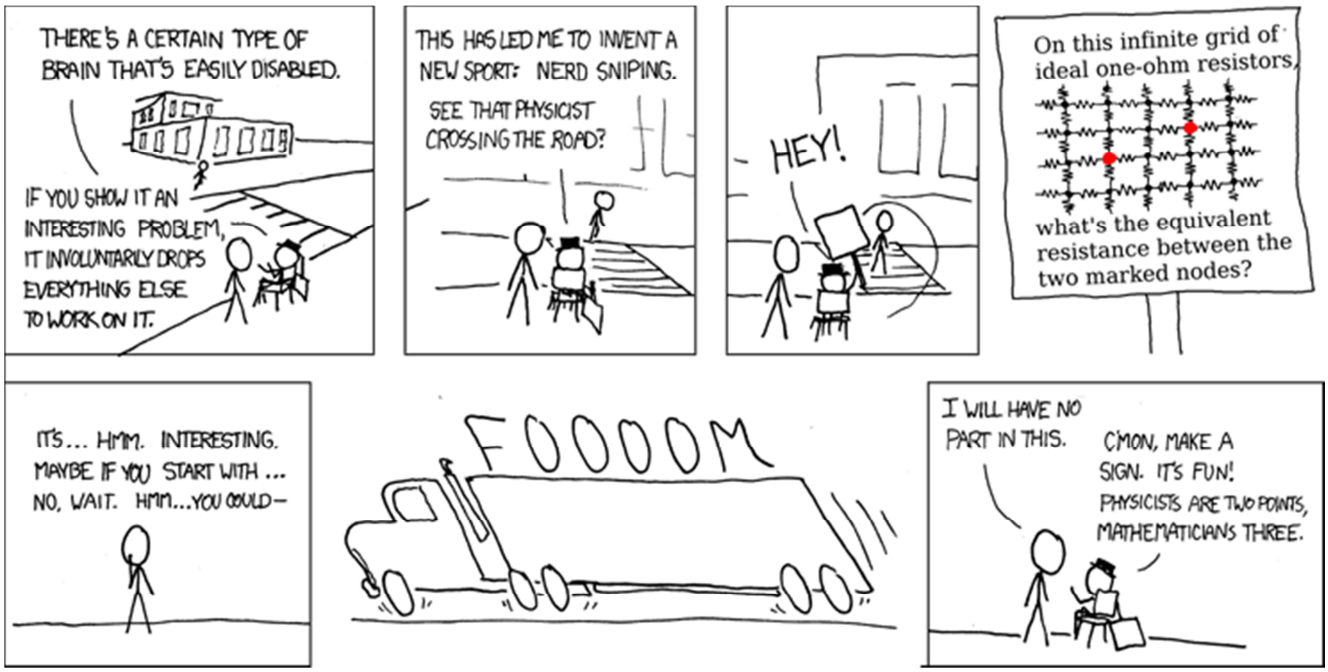
Name (Last, First) Solution

printed

Pledge: On my honor, I have neither given nor received unauthorized aid on this examination.

_____

signed


THERE'S A CERTAIN TYPE OF BRAIN THAT'S EASILY DISABLED.
IF YOU SHOW IT AN INTERESTING PROBLEM, IT INVOLUNTARILY DROPS EVERYTHING ELSE TO WORK ON IT.
THIS HAS LED ME TO INVENT A NEW SPORT: NERD SNIPING.
SEE THAT PHYSICIST CROSSING THE ROAD?
HEY!
On this infinite grid of ideal one-ohm resistors,
what's the equivalent resistance between the two marked nodes?
IT'S... HMM. INTERESTING. MAYBE IF YOU START WITH ... NO, WAIT. HMM...YOU COULD—
FOOOOM
I WILL HAVE NO PART IN THIS.
C'MON, MAKE A SIGN. IT'S FUN! PHYSICISTS ARE TWO POINTS, MATHEMATICIANS THREE.

xkcd.org

1. [20 points] The Channel Forty-two weather team reported the temperatures in Bleaksburg at noon for each of the last five days, rounded to the nearest degree. The five temperatures were all different, and the product of the temperatures was, curiously, equal to 12. Can you determine what those five temperatures must have been (although not their order), or is there not enough information, or are the given facts impossible?

The factors of 12 are -12, -6, -4, -3, -2, -1, 1, 2, 3, 4, 6 and 12.

Since we need five numbers, we can rule out -12, -6, 6 and 12 immediately, since any of those would require that we write 1 or 2 as a product of four of the remaining candidates, and that's impossible.

And, we can rule out -4 and 4 for the same reason, since there's no way to write 3 as a product of four of the remaining candidates.

So, we're down to: -3, -2, -1, 1, 2 and 3. Now, we must use -3 or 3, since no combination could give us a product of 12 if we don't. And, that means we need both -2 and 2, since there's otherwise no way to get a product of 12. But we now have (-3, -2, 2) or (-2, 2, 3), which gives us a product of 12 or -12. So, we must use both -1 and 1 to get to five factors.

And, we need an even number of negative values to get a positive product, so we cannot use -3.

Therefore, the only solution is that the temperatures were: -2, -1, 1, 2 and 3.

2. [20 points] In a foreign language *ho lew trj* means "buy that dog", *lew ra* means "dog food" and *gi trj nk* means "that green car". Which words would be used to say "buy food"? Why?

Since "ho lew trj" and "lew ra" must both contain a word for "dog", and the only foreign word they have in common is "lew", we know that "lew" must mean "dog".

And, that tells us that "ra" must mean "food".

Now, "ho lew trj" and "gi trj nk" must both contain a word meaning "that", and the only candidate is "trj", so "trj" must mean "that".

But then, in "ho lew trj", "ho" must mean "buy" since the other two words have been determined to mean something else.

Therefore, "buy food" must be indicated by "ho ra" or "ra ho".

(It could be either, since we don't know anything about the grammar of this language.)

3. [20 points] Two ranchers sold a herd of cattle and received as many dollars for each animal as there were cattle in the herd. (If there were two animals, they received two dollars for each one; if three, three dollars for each one; etc. We're told that they started with no more than 15 cattle.) With the money, they bought as many sheep as they could at 10 dollars per head, and a goat with the remainder (less than 10 dollars). Finally, they divided the animals between them. There was, however, an odd number of sheep. So one rancher, who got the goat, was given his friend's new pocketknife as compensation. If this was an equal division, what was the value of the pocketknife? Is it possible to say anything precise about how many cattle the two ranchers sold? If yes, what?

Let C be the number of cattle they sold. Then they got $C for each one, so the total amount of money they got must have been $C^2.

Since C is between 1 and 15, C^2 has to be one of the following values:

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225

Now what? Well, they paid $10 for each sheep, and they bought an odd number of sheep… so if we divide the amount of money they received by 10, we must get an odd number, with something left over to pay for the goat.

Looking at our list of candidates for C^2, there are only three values that would yield an odd result when divided by 10: 16, 36, and 196.

So, they must have gotten either $16, $36 or $196 for the cattle, but we can't say which. (Therefore, we can say the number of cattle was either 4 or 6 or 14, but we can't narrow it down further.)

However, either way, they would have had $6 left over to pay for the goat. And that would mean that whoever got the goat had received $4 less value than the other rancher.

So, it might seem the pocketknife must have been worth $4. But… that doesn't take into account that whoever gave up the pocketknife LOST the value of the knife.

So, the pocketknife must have been worth $2.

4. [20 points] Four lawn care fanatics, Fred, John, Alice and Nina between them own 17 grass cutting devices consisting of shears, string trimmers, and power mowers. Fred likes to be accurate, so he owns 4 shears but no string trimmers. John is just the opposite; he prefers speed and has a small lawn, and therefore his implements include 2 shears and 3 mowers. Alice's equipment includes 2 string trimmers and 1 power mower. Nina, a woman of simple tastes, has a total of 3 implements, among which is 1 shears, but 0 string trimmers. Altogether, the four own 7 shears and 7 mowers, and a total of 17 implements.

Note: a good, well-labeled externalization for this problem will be sufficient to explain your logic.

	Shears	String trimmers	Power mowers	Totals
Fred	4	0	1_7	5_6
John	2	1_2	3	6_3
Alice	0_4	2	1	3_5
Nina	1	0	2_8	3
Totals	7	3_1	7	17

Explanation:

1: from given row values, this must be 17 – 7 – 7
2: from known column values, this must be 3 – 2
3: from known row values, this must be 2 + 1 + 3
4: from known column values, this must be 7 – 1 – 2 – 4
5: from known row values, this must be 0 + 2 + 1
6: from known column values, this must be 17 – 6 – 3 – 3
7: from known row values, this must be 5 – 4 – 0
8: from known row values, this must be 3 – 1 – 0 (or use known column values)

5. [20 points] Four men were asked about their yearly incomes. Their names were Earl, Moe, Luis, and Randy and their professions are architect, carpenter, plumber, and mason (not necessarily in that order). Each made two statements, but the only statements whose correctness can be depended on are those in which the speaker specifically mentions his own profession. Other statements may or may not be true.

Earl: The plumber makes three times as much as the carpenter. The architect makes more money than I do.

Moe: The carpenter makes more money than the plumber. Luis is either the mason or the architect.

Luis: I make more than the architect. The carpenter makes less than each of the others.

Randy: The plumber makes twice as much as the carpenter. I make more than the mason.

Match each person with his profession.

Note: For this problem, explain all your inferences carefully. Every conclusion you reach must be justified by referring to the statements given above, or other facts you've inferred from them. Be precise and complete. Use externalization.

Label the eight assertions E1, E2, M1, M2, L1, L2, R1 and R2.

Now, Earl cannot be the architect; if he were, then E2 would imply that Earl makes more than he makes.

Similarly, from L1, Luis implies he is not the architect either. And, from R2, Randy implies he is not the mason.

Now it gets subtle. Note that E1 and M1 directly contradict each other, so they cannot both be true. Likewise, M1 and L1 directly contradict each other. And, M1 and R1 also directly contradict each other; as to E1 and R1. But this doesn't immediately tell us anything...

Suppose, say, that Earl is the carpenter. Then E1 must be true (since Earl mentions his profession), and therefore M1 must be false. That implies that Moe is neither the carpenter nor the plumber; otherwise, M1 must be true. And, R1 must also be false, hence Randy cannot be the plumber. This implies, by elimination, that Luis must be the plumber. But, that implies that M2 is false, which would mean that Moe cannot be either the mason or the architect. That doesn't leave any possible occupation for Moe, a contradiction. Therefore, Earl cannot be the carpenter.

If Earl is not the carpenter, then E1 must be false, so Earl cannot be the plumber either. And, we already know Earl cannot be the architect, therefore, Earl must be the mason.

So, M2 is false (we've ruled out Luis being either the mason (Earl is) or the architect. But that implies that Moe cannot be either the mason or the architect, since if he were M2 would have to be true. So, by elimination, Randy must be the architect.

Now, Moe must be either the carpenter or the plumber, which means that M1 must be true. But M1 contradicts L2, so L2 is false, and Luis cannot be the carpenter.

So, by elimination, Luis must be the plumber and Moe must be the carpenter.

Some notes on the logic of the problem:

When Earl says "the architect makes more than I do", that does imply Earl cannot be the architect (since, if so he'd be saying he makes more than he makes, which is nonsense). But that doesn't tell us whether the architect does, in fact, make more money than Earl. Either would be consistent with what we know at this point.

When Moe says "Luis is either the mason or the architect", that doesn't imply any definite conclusions (by itself). If Moe is neither the mason nor the architect, then the statement could be either true or false. If Moe is actually the mason or the architect, then the statement must be true, and that would tell us what Luis is, assuming we knew which Moe was. But we do not (initially) know whether Moe is the mason or the architect or something else, so his statement is not useful (initially).

In the end, what do we know about how much they earn?

Since Moe is the carpenter, it must be true that the carpenter makes more than the plumber.

Since Earl is the mason, his statement about how his earnings compare to the architect tells us nothing, because that statement does not mention his profession.

Similarly Luis is the plumber, so his statement about how his earnings compare to the architect also tells us nothing.

And, similarly, Randy's statements about earnings also tell us nothing, since neither mentions his own profession.