Inductive Reasoning

Strictly speaking, all our knowledge outside mathematics… consists of conjectures.

…

There are, of course, conjectures and conjectures. There are highly respectable and reliable conjectures as those expressed in certain general laws of physical science.

There are other conjectures, neither reliable nor respectable, some of which may make you angry when you read them in a newspaper.

And in between there are all sorts of conjectures, hunches, and guesses.

George Polya

Inducing a Hypothesis

Inductive reasoning draws generalized conclusions from a finite collection of specific observations.

The act of inducing a conclusion should provide some degree of support for the truth of the conclusion, but cannot ensure it.

Specific observation:

Every student I have encountered at Virginia Tech is a hominid.

Generalized conclusion:

All students at Virginia Tech are hominids.

The conclusion may be strengthened by an indication of how long I have been at Virginia Tech and how many students I have encountered there.

Nevertheless… the conclusion cannot be absolutely affirmed.

Some Kinds of Inductive Reasoning

Simple Induction

reason from observations of a sample group to a conclusion about a member of the larger population

17 of 20 dentists we surveyed majored in Mining Eng as undergrads.

So, there is a high probability, around 85%, that your dentist will have majored in Mining Eng as an undergrad.

This is related to *inductive generalization*, which would conclude from the same evidence that:

So, 85% of all dentists majored in Mining Eng as undergrads.

The Problem of Induction

Does inductive reasoning lead to knowledge?

What justifies either of the following:

Generalizing about the properties of a class of objects based on some number of observations of particular instances of that class.

For example, the inference that "all swans we have seen are white, and therefore all swans are white," before the discovery of black swans.

Presupposing that a sequence of events in the future will occur as it always has in the past.

For example, that the laws of physics will hold as they have always been observed to hold.

The Problem of Induction

The problems just considered have a hallowed history in Western philosophy, from the Greeks to David Hume and beyond…

In the end, perhaps Hume put it best:

> If we be, therefore, engaged by arguments to put trust in past experience, and make it the standard of our future judgement, these arguments must be probable only, or such as regard matter of fact and real existence, according to the division above mentioned. But that there is no argument of this kind, must appear, if our explication of that species of reasoning be admitted as solid and satisfactory.
>
> We have said that all arguments concerning existence are founded on the relation of cause and effect; that our knowledge of that relation is derived entirely from experience; and that all our experimental conclusions proceed upon the supposition that the future will be conformable to the past.
>
> To endeavour, therefore, the proof of this last supposition by probable arguments, or arguments regarding existence, must be evidently going in a circle, and taking that for granted, which is the very point in question.

An Enquiry Concerning Human Understanding

The Necessity of Induction

So, does inductive reasoning lead to knowledge?

In some instances, perhaps… and in other instances, perhaps not.

However, there seems no alternative if we are to form any conclusions regarding real phenomena.

We may observe and induce, and maintain eternal vigilance regarding the correctness of the conclusions we induce.

Sequence Example

A common test question requiring induction is to determine the next value in an incomplete sequence of integers. For example:

11, 13, 17, 19, ??

What would the next value be?

23?

31?

Something else?

The key to induction is to look for patterns… sometimes there is no obvious pattern… and even an obvious pattern is never unique.

Bachet's Conjecture

Doodling with small integers, you might notice that:

1	1
2	1 + 1
3	1 + 1 + 1
4	4
5	4 + 1
6	4 + 1 + 1
7	4 + 1 + 1 + 1
8	4 + 4
9	9
10	9 + 1

So what? Well, so far, every positive integer we've looked at can be written as a sum of perfect squares, and we've never needed more than four perfect squares.

Turns out the conjecture holds up if you continue the table…

Bachet's Conjecture

One more:

325 289 + 16 + 16 + 4

Fine… so we might (if we were Claude Gaspard Bachet de Méziriac) conjecture that:

Any positive integer is a square, or the sum of two, three, or four squares.

And… if you were Joseph Louis Lagrange, you might <u>prove</u> it.

Euler's Formula

Leonhard Euler noticed that if we define

$$p(n) = n^2 - n + 41$$

then the values $p(1)$, $p(2)$, …, $p(20)$ are

41, 43, 47, 53, 61, 71, 83, 97, 113, 131, 151,
173, 197, 223, 251, 281, 313, 347, 383, 421

all of which are prime.

So, you might conjecture…

And… if you consider $p(41)$ you will realize you are wrong…

Final Note

Do not confuse inductive reasoning with mathematical induction.

Despite some superficial similarities, they are fundamentally different.