



READ THIS NOW!

- Print your name in the space provided below.
- There are 5 short-answer questions, priced as marked. The maximum score is 100.
- The grading of each question will take into account whether you obtained a correct solution and how well you presented your analysis and justified your logic. In most cases, as much weight will be given to the presentation and explanation of your analysis as to whether the solution is fully correct. Legibility will be strongly considered in the grading. You may use scratch paper to work out your solution before finalizing it on the exam.
- Externalize! Whether it's a drawing, a table, an equation or something else, externalize! And make the externalization explicit in your answer! Label things for clarity!
- You may use the supplied extra paper for scratch work. Write your name on any scratch work sheets you use and turn those in with your exam.
- All final answers must be written on the test form itself.
- When you have finished, sign the pledge at the bottom of this page and turn in the test.
- This examination is closed book and closed notes, aside from the permitted one-page formula sheet. No calculators or other computing devices may be used. The use of any such device will be interpreted as an indication that you are finished with the test and your test form will be collected immediately.
- Until solutions are posted, you may not discuss this examination with any student who has not taken it.
- Failure to adhere to any of these restrictions is an Honor Code violation.

Name (Last, First) **Solution**

printed

Pledge: On my honor, I have neither given nor received unauthorized aid on this examination.

_____ *signed*

1. [20 points] At one time, miners were paid in "private" money called scrip that could only be spent at the mining company's stores and other facilities.

In 1910, miners working for the Deep Delvings Coal Company were paid \$0.50 (in scrip) for each ton of coal they brought out of the mine. The average miner was paid \$100 per week (in scrip). The Company offered the miners a choice between two proposals:

- i) Each miner would now be paid 20% more (in scrip) for each ton of coal he mined, and prices in the Company stores and other facilities would remain the same.
- ii) The rate paid per ton of coal would stay the same, but the prices in the Company stores and other facilities would all be reduced by 20%.

Which of the plans would be more advantageous for the miners? Or would the two plans be equivalent? Justify your conclusion carefully.

Under the current scheme, the average miner earns \$100 per week (in scrip). Let's call this version 0, then we could say that the average miner can buy stuff worth \$100 in version 0 scrip per week.

Now, under option i, each miner would be paid \$120 per week (in scrip), and since the prices remain the same, the average miner could buy stuff worth \$120 in version 0 scrip.

And, under option ii, each miner would still be paid \$100 per week (in scrip), but now the prices are lower and so the average miner could buy stuff worth $\$100/0.80$ in version scrip. Now, $\$100/0.80 = \125 in version 0 scrip.

So, option ii is the better option (although it's not staggeringly better).

Notes:

- Some students suggested the possibility that a miner could mine more (or less) than the average of 200 tons of coal per week. That's true, but it is independent of this question. The two options alter the purchasing power of the average miner's income, and that effect would be separate from the effect of changing productivity.
- A common error was to overlook the fact that \$1 of scrip under option ii would be worth more than \$1 of scrip under option i; this usually manifested via an argument that concluded something like "a miner would have the same amount of money left over" under either plan (and under some assumptions about how the money was spent). But the same amount, in \$'s of scrip, doesn't have the same purchasing power under the two plans.

- 2. [20 points] A florist has four kinds of fresh flowers available for bouquets. Three customers (Dwight, Robert and Joe) each ordered bouquets. Dwight ordered a bouquet of 10 flowers, including 2 roses and 3 anemones. Robert ordered a bouquet that included 4 gladioli and 2 carnations. Joe's bouquet included 1 anemone and 5 carnations, and had a total of 8 flowers.

Altogether the three ordered a total of 4 roses, 4 anemones and 9 gladioli, and a total of 25 flowers.

How many flowers of each type did the three men order?

Note: a good, well-labeled externalization for this problem will serve as an explanation of your logic.

This is best externalized via a table; here is mine:

Name	# roses	# anemones	# gladioli	# carnations	totals:
Dwight	2	3	4	1	10
Robert	1	0	4	2	7
Joe	1	1	1	5	8
totals:	4	4	9	8	25

The values entered in green are given directly in the problem statement. The values entered in red are inferred as described below:

We can immediately see that Robert ordered 0 anemones, and that the total number of carnations must be 8.

Then, we see that Robert must have bought a total of 7 flowers, and that gives us that he bought 1 rose.

We also see that Joe must have bought 1 rose, and therefore that he must have bought 1 gladiolus.

Then it's obvious that Dwight bought 4 gladioli and 1 carnation.

- [20 points] A courier must make a trip from Asheville to Donegal, passing through Bakerton and Crestburg along the way (in that order). It is ten miles further from Asheville to Bakerton than from Bakerton to Crestburg, and it is 20 miles further from Bakerton to Crestburg than from Crestburg to Donegal. She finds the total distance from Asheville to Donegal is 200 miles.

How far is it from Asheville to Crestburg?

Externalize carefully! Use labels and give good descriptions.

I'll first externalize with a line diagram to visualize the relative locations, and then I'll label the segments:



Now, from the given information, I see that we have the following relationships:

$$\begin{aligned}
 X &= Y + 10 \\
 Y &= Z + 20 \\
 X + Y + Z &= 240
 \end{aligned}$$

And, these are easily solved; substituting yields:

$$X + (X - 10) + (X - 10 - 20) = 200$$

$$3X - 40 = 200$$

$$3X = 240$$

$$X = 80, \text{ so } Y = 70, \text{ and } Z = 50$$

Then, the distance from Asheville to Crestburg is $X + Y$ or 150 miles.

4. [20 points] Consider the following argument:

If walnut trees do not bear nuts, then bears live in walnut trees.
 Bears do not live in walnut trees.
 If bears live in walnut trees, then squirrels do not climb walnut trees.
 Therefore, squirrels climb walnut trees.

Is the argument valid? Justify your conclusion carefully, according to the definition of what makes an argument valid.

An argument is valid if the form of the argument is a tautology. We can symbolize the given argument in several ways; I'll do it this way:

W: walnut trees bear nuts
 B: bears live in walnut trees
 S: squirrels climb walnut trees

(I decided to make the symbols represent "positive" statements. That was just a matter of taste.)

Then the argument can be symbolized as

if (if not W then B) and (not B) and (if B then not S), then S

Is this a tautology? If not, there's a case in which all the premises are true but the conclusion is false. Now, that would require that S is false. If S is false, then not S is true, and the third premise will be true whether B is true or false. The second premise will only be true if B is false. And then, the first premise will only be true if not W is false, or W is true.

So, we have a case in which the form of the argument is not a tautology, so it's not valid.

We could also construct the entire truth table, but that's tedious.

Notes:

- Some answers focused on whether the premises or the conclusion of the argument were true. That's irrelevant to whether the argument is valid.

5. [20 points] Four colleagues were feeling adventurous on Friday evening, when they went out for their customary weekly dinner together, and decided to go to the Excellent Table for some Ethiopian food. After poring over the exotic menu, each ordered a different main course (including mixed vegetables). Getting into the spirit of things, two of the four ordered beverages they had never heard of, Eritrean honey wine and pineapple soda; but the other two stuck with their usual beverages. From the clues below, determine both the main course and the beverage that each ordered that evening:
- The lentil stew was eaten by a woman.
 - Alexey did not order the chicken stew.
 - Sean drank the honey wine.
 - The people who ordered pineapple soda and red wine are not of the same gender.
 - Deborah ordered the spicy greens.
 - Daphne did not order either pineapple soda or sparkling soda.

Note: For this problem, explain all your inferences carefully. Every conclusion you reach should be justified. Be precise and complete.

There are four main courses: mixed vegetables, spicy greens, lentil stew, chicken stew
 There are four beverages: Ethiopian honey wine, red wine, sparkling soda, pineapple soda

The facts can be organized in a simple table; the facts that are directly given in the clues can be added to the table:

- Sean drank the Eritrean honey wine (clue c)
- Deborah ordered spicy greens (clue e)

Name	main course	beverage
Alexey		
Sean		honey wine
Deborah	spicy greens	
Daphne		

Now, clue a tells us that the lentil stew was eaten by a woman. Assuming that the names are used in their usual ways, Alexey is male and Daphne is female, and Sean could be either.

So, Alexey did not eat lentil stew, and by clue b he did not eat chicken stew; therefore by elimination, Alexey must have eaten mixed vegetables.

If Sean is male, then he ate chicken stew and Daphne ate lentil stew; if Sean is female, she ate chicken or lentil stew, and Daphne ate the opposite. I assume there is a unique solution, so I'll opt for the former.

Name	main course	beverage
Alexey	mixed vegetables	
Sean	chicken stew	honey wine
Deborah	spicy greens	
Daphne	lentil stew	

From clue f, Daphne must have had the red wine.

Name	main course	beverage
Alexey	mixed vegetables	
Sean	chicken stew	honey wine
Deborah	spicy greens	
Daphne	lentil stew	red wine

From clue d, the pineapple soda was ordered by a male, so that was Alexey, and therefore Deborah must have had the sparkling soda:

Name	main course	beverage
Alexey	mixed vegetables	pineapple soda
Sean	chicken stew	honey wine
Deborah	spicy greens	sparkling soda
Daphne	lentil stew	red wine

Notes:

- A significant number of students overlooked given information. For example, it was surprisingly common to see the claim that only three main courses were specified. The moral of that is that you must read the problem carefully.
- Some students were confused by the statement about people drinking their usual beverage vs people drinking something new. That statement was superfluous and had no effect on the analysis. But... it is possible to say, in the end, who drank his/her usual beverage and who did not.