This is a purely individual assignment. Prepare your answers to the following questions <u>in a plain ASCII text file or a Word document</u>. No other formats will be graded. Submit your file to the Curator system by the posted deadline for this assignment. No late submissions will be accepted without special permission from your instructor.

You will submit your answers to the Curator System (www.cs.vt.edu/curator) under the heading `OOC03`.

1. [10 points] Consider the following (alleged) inference rule:

    If A, then B.
    B.
    Therefore, A.

    Is this actually an inference rule? Verify your conclusion by constructing a relevant truth table.

2. [10 points] Suppose an argument was constructed using the (alleged) inference rule given in question 1, by substituting propositions for the variables A and B.

    a) Without knowing what specific propositions are substituted for A and B, is it possible to say anything definitive regarding the <u>validity</u> of the argument? Explain clearly.

    b) Again, without knowing what specific propositions are substituted for A and B, is it possible to say anything definitive regarding the <u>soundness</u> of the argument? Explain clearly.

3. [25 points] Consider each of the following arguments, and decide whether the argument is invalid or valid. For each argument, justify your conclusion clearly.

    a) If a mouse has built a nest in my car's heater, then I will have a cold drive to work. I was warm during my drive to work. Therefore, a mouse has not built a nest in my car's heater.

    b) If a mouse has built a nest in my car's heater, then I will have a cold drive to work. I was cold during my drive to work. Therefore, a mouse has not built a nest in my car's heater.

    c) If it snows tonight, then I will walk to campus in the morning. If I walk to campus in the morning, I will have to skip breakfast. So, if it snows tonight then I will skip breakfast in the morning.

    d) If it snows tonight, then I will walk to campus in the morning. If I walk to campus in the morning, I will have to skip breakfast. So, if I eat breakfast in the morning then it must have not snowed overnight.

    e) If it snows tonight, then I will walk to campus in the morning. If I walk to campus in the morning, I will have to skip breakfast. So, if it does not snow tonight then I will eat breakfast in the morning.

4. [25 points] Consider each of the following arguments, and decide whether the argument is invalid or valid but unsound or sound. For each argument, justify your conclusion clearly.

    a) If Yucca Mountain is in Nevada, then it is west of the Mississippi River. Yucca Mountain is not west of the Mississippi River. Therefore, Yucca Mountain is not in Nevada.

    b) If Yucca Mountain is in Nevada, then it is west of the Mississippi River. Yucca Mountain is west of the Mississippi River. Therefore, Yucca Mountain is in Nevada.

    c) Yucca Mountain is in Nevada or Mount Erebus is north of the Arctic Circle. Yucca Mountain is in Nevada. Therefore, Mount Erebus is not north of the Arctic Circle.

d) Yucca Mountain is in Nevada or Mount Erebus is north of the Arctic Circle. Mount Erebus is not north of the Arctic Circle. Therefore, Yucca Mountain is in Nevada.

e) Mount Erebus is north of the Arctic Circle. Mount Erebus is not north of the Arctic Circle. Therefore, Yucca Mountain is not in Nevada.

The following definitions are useful for the remaining questions:

If x and y are integers, and there is an integer q such that x = y*q, we say that y is a *divisor* of x, and if y > 0 we say that y is a *positive divisor* of x. For example, 3 is a divisor of -87 because -87 = 3 *- 29.

If x is an integer, and y is a positive divisor of x and y is less than |x|, we say that y is a *proper divisor* of x. For example, the proper divisors of 12 are 1, 2, 3, 4 and 6.

If x is an integer greater than 1, and the only proper divisor of x is 1, we say that x is *prime*.

5. [10 points] Is the following statement true or false? Justify your conclusion carefully.

Some integer is equal to the sum of its proper divisors.

6. [10 points] Is the following statement true or false? Justify your conclusion carefully.

Every integer x has a divisor y such that y < x.

7. [10 points] Is the following statement true or false? Justify your conclusion carefully.

Some prime integer is equal to the sum of its positive divisors.