You may work in pairs or purely individually for this assignment. Prepare your answers to the following questions in a plain ASCII text file or MS Word document. Submit your file to the Curator system by the posted deadline for this assignment. No late submissions will be accepted. If you work in pairs, list the names and email PIDs of both members at the beginning of the file, and submit your solution under only one PID. No other formats will be graded.

For this assignment, you may (and are encouraged to) work in pairs; if you do so, you must also write your solutions in such a way that it is clear how each member contributed to deriving the solution.

You will submit your answers to the Curator System (www.cs.vt.edu/curator) under the heading `OOC06`.

For the questions on probability, express your answer first in terms of appropriate notation (factorials, combinations, permutations), and then **for parts 1a - 1c** simplify your answer to a single decimal value, rounded to two digits after the decimal point. Do not simplify your answers for parts **1d – 1e**.

1. An urn contains 8 red balls, 6 blue balls, 7 green balls, and 10 yellow balls.

a) [15 points] If two balls are drawn randomly from the urn, what is the probability that they will be of the same color?

There are four cases in which two balls of the same color are drawn, because there are four colors and at least two balls are available in each color. Since you could have two red or two blue or two green or two yellow balls, the addition rule applies. The number of different ways to get two balls of the same color would be:

$$\binom{8}{2}+\binom{6}{2}+\binom{7}{2}+\binom{10}{2}=28+15+21+45=109$$

There are 31 balls in the urn, so the number of ways to draw two balls is just:

$$\binom{31}{2}=465$$

So, the probability of getting two balls of the same color would be:

$$\frac{\binom{8}{2}+\binom{6}{2}+\binom{7}{2}+\binom{10}{2}}{\binom{31}{2}}=\frac{109}{465}\approx 0.23$$

b) [15 points] If six balls are drawn randomly from the urn, what is the probability that they will be of six different colors?

Since there are only four colors available, the probability of getting six different colors would be 0.

c) [15 points] If six balls are drawn randomly from the urn, what is the probability there will be two balls of one color and four balls of another color?

The tempting way to solve this is to imagine choosing the color for the pair, then choosing two balls of that color, then choosing a different color for the four, and finally choosing four balls of that color. But we have different numbers of the different colors, so we cannot say in how many ways we can pick two balls without knowing which color is involved.

Therefore, we must consider this tediously. We could make the following choices:

red and blue:	2R4B or 4R2B
red and green:	2R4G or 4R2G
red and yellow:	2R4Y or 4R2Y
blue and green:	2B4G or 4B2G
blue and yellow:	2B4Y or 4B2Y
green and yellow:	2G4Y or 4G2Y

So, the number of ways to pick two balls of one color and four of another would be:

$$\binom{8}{2}\binom{6}{4}+\binom{6}{2}\binom{8}{4}+\binom{8}{2}\binom{7}{4}+\binom{7}{2}\binom{8}{4}+\binom{8}{2}\binom{10}{4}+\binom{10}{2}\binom{8}{4}+\binom{6}{2}\binom{7}{4}+\binom{7}{2}\binom{6}{4}+\binom{6}{2}\binom{10}{4}+\binom{10}{2}\binom{6}{4}+\binom{7}{2}\binom{10}{4}+\binom{10}{2}\binom{7}{4}=$$
$$28\cdot 15+15\cdot 70+28\cdot 35+21\cdot 70+28\cdot 210+45\cdot 70+15\cdot 35+21\cdot 15+15\cdot 210+45\cdot 15+21\cdot 210+45\cdot 35=$$

So, the probability of getting 2 balls of one color and 4 balls of another color would be:

$$\frac{\binom{8}{2}\binom{6}{4}+\binom{6}{2}\binom{8}{4}+\binom{8}{2}\binom{7}{4}+\binom{7}{2}\binom{8}{4}+\binom{8}{2}\binom{10}{4}+\binom{10}{2}\binom{8}{4}+\binom{6}{2}\binom{7}{4}+\binom{7}{2}\binom{6}{4}+\binom{6}{2}\binom{10}{4}+\binom{10}{2}\binom{6}{4}+\binom{7}{2}\binom{10}{4}+\binom{10}{2}\binom{7}{4}}{\binom{31}{6}}$$

d) [10 points] If three balls are drawn from the urn, and all of them are yellow, what is the probability that the next ball drawn will also be yellow?

If three yellow balls have been drawn from the urn, there are 6 blue balls left, and 28 altogether. So the probability the next ball would be blue is:

$$\frac{\binom{6}{1}}{\binom{28}{1}}=\frac{6}{28}\simeq 0.214$$

e) [10 points] If three balls are drawn from the urn, and all of them are yellow, what is the probability that the next ball drawn will be blue?

If three yellow balls have been drawn from the urn, there are 7 yellow balls left, and 28 altogether. So the probability the next ball would be yellow is:

$$\frac{\binom{7}{1}}{\binom{28}{1}} = \frac{7}{28} \simeq 0.25$$

2. Consider randomly selecting cards from a standard 52-card poker deck.

a) [10 points] In how many different ways can you construct a 5-card hand so that the hand contains 2 cards of one value and 3 cards of different values than all the other cards in the hand?

We can count this by considering the following steps:

- **pick a value for the pair**
- **pick two cards of that value**
- **pick three different values for the other cards**
- **pick one card of each of those values**

So, the number of ways to do this would be:

$$\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}\binom{4}{1}\binom{4}{1} = 13 \cdot 6 \cdot 220 \cdot 4 \cdot 4 \cdot 4 = 1098240$$

b) [10 points] In how many different ways can you construct a 7-card hand so that the hand contains 2 cards of one value, 4 cards of another value, and a single card of a different value than all the other cards in the hand?

We can count this by considering the following steps:

- **pick a value for the pair**
- **pick two cards of that value**
- **pick a value for the 4-some**
- **pick four cards of that value**
- **pick a value for the singleton**
- **pick one card of that value**

So, the number of ways to do this would be:

$$\binom{13}{1}\binom{4}{2}\binom{12}{1}\binom{4}{4}\binom{11}{1}\binom{4}{1} = 13 \cdot 6 \cdot 12 \cdot 1 \cdot 11 \cdot 4 = 41184$$

3. [15 points] A coin is tossed 100 times. The coin is "fair" in the sense that on any single toss it is equally likely to land heads-up or tails-up. What is the probability that the coin will land heads-up exactly 50 times?

For credit, you must explain carefully the logic you used to derive your answer.

If we flip the coin 100 times and it comes up heads 50 times, in how many different ways can we get 50 heads? That would be the number of ways to choose on which 50 flips the coin landed heads-up, which would just be:

$$\binom{100}{50}$$

How many different results could we get if we flip the coin 100 times? That's simpler, since there are 2 possible outcomes each time we flip the coin, there would be 2^{100} different possible sequences of outcomes.

So the probability of getting exactly 50 heads would be:

$$\frac{\binom{100}{50}}{2^{100}}$$

That's not very informative though... what's the value? About 0.08.

Note: there's a handy calculator for binomial trials (two possible outcomes) at

http://stattrek.com/online-calculator/binomial.aspx