You may work in pairs or purely individually for this assignment. Prepare your answers to the following questions in a plain ASCII text file or MS Word document. Submit your file to the Curator system by the posted deadline for this assignment. No late submissions will be accepted. If you work in pairs, list the names and email PIDs of both members at the beginning of the file, and submit your solution under only one PID. No other formats will be graded.

For this assignment, you may (and are encouraged to) work in pairs; if you do so, you must also write your solutions in such a way that it is clear how each member contributed to deriving the solution.

You will submit your answers to the Curator System (www.cs.vt.edu/curator) under the heading `OOC05`.

For each of the following questions, you are asked to derive a recurrence relation. You must explain clearly why your recurrence relation is correct, but you are not required to solve the recurrence.

1. [25 points] When climbing a flight of stairs, you can take one stair at a time or take two stairs at a time. Of course, you can mix the two choices as well. Find a recurrence relation for the number of different ways you could climb a flight of N stairs, where $N > 0$.

Let C(N) be the number of ways to climb a flight of N stairs, where N > 0.

Now it's obvious enough that C(1) = 1 (1) and C(2) = 2 (11 or 2).

For C(3): 111 21 12

For C(4): 111 211 121 112 22

So, the pattern is this... given a flight of N stairs, your last effort can either be a single stair, or two stairs at once. If your last effort is a single stair, that was preceded by one of the solutions for a flight of N – 1 stairs. If your last effort is two stairs at once, that preceded by one of the solutions for a flight of N – 2 stairs.

Therefore, the recurrence is:

$$C_1 = 1, C_2 = 2$$
$$C_N = C_{N-1} + C_{N-2} \text{ for } N > 2$$

2. [25 points] A robot can move forward by stepping 10 cm, or by hopping 20 cm, or by leaping 30 cm. Find a recurrence relation for the number of different ways the robot could move forward 10*N* cm, for $N > 0$.

Let M(N) be the number of ways the robot can move forward a distance of 10N cm, for N > 0.

Obviously, M(1) = 1 since the robot could only take a 10cm step.

And, M(2) = 2 since the robot could either take two 10cm steps or hop 20cm.

And, M(3) = ?

Well, the robot has these options:

step step step
step hop
hop step
leap

So M(3) = 4.

For N = 4, we could either take a solution for N = 3 and append a step, take a solution for N = 2 and append a hop, or take a solution for N = 1 and append a leap... without producing any solutions for N = 4 twice.

Let's check; the robot has these options:

step step step step
step hop step
step step hop
hop step step
hop hop
step leap
leap step

So M(4) = 7.

So, we have the recurrence:

$$M_1 = 1, M_2 = 2, M_3 = 4$$
$$M_N = M_{N-1} + M_{N-2} + M_{N-3} \text{ for } N > 3$$

3. [25 points] Find a recurrence relation for the number of ways to arrange a sequence of flags on an *N*-foot flagpole, if you have three kinds of flags: blue flags and green flags that are 3 feet tall and red flags that are 1 foot tall.

For N = 1:	R	
For N = 2:	RR	
For N = 3:	RRR	
	B G	
For N = 4:	RRRR BR GR	from N = 3, append R
	RG RB	from N = 1, append G or B
For N = 5:	RRRRR BRR GRR RGR RBR	from N = 4, append R
	RRG RRB	from N = 2, append G or B

So, we see a relationship and a recurrence:

$$F_1 = 1, F_2 = 1, F_3 = 3$$
$$F_N = F_{N-1} + 2F_{N-3} \text{ for } N > 3$$

4. [25 points] You have an unlimited number of red, blue and green cubes. Find a recurrence relation for the number of different ways to build a vertical stack of N blocks, for $N > 0$, such that there are never two adjacent blue blocks.

Let S_N be the number of ways to build a stack of N blocks, so that no blue blocks are adjacent.

Obviously, S_1 is 3: R B G

And, S_2 is 8: RR BR GR RG BG GG RB GB

For N = 3, we have: RRR BRR GRR RGR BGR GGR RBR GBR

RRG BRG GRG RGG BGG GGG RBG GBG

RRB BRB GRB

RGB BGB GGB

Now, I can append R or G to any solution for N = 2 and obtain a valid solution for N = 3; that gives us $2S_2$ solutions for N = 3.

And, I can append RB or GB to any solution for N = 1 and obtain a valid solution for N = 3; that gives us $2S_1$ solutions for N = 3.

And, neither of those approaches will yield the same solution for N = 3 (note the differences in the final block being added).

So, I get the following recurrence:

$$S_N = \begin{cases} 3 & N = 1 \\ 8 & N = 2 \\ 2F_{N-1} + 2F_{N-2} & N > 2 \end{cases}$$