

You may work in pairs or purely individually for this assignment. Prepare your answers to the following questions in a plain ASCII text file or MS Word document. Submit your file to the Curator system by the posted deadline for this assignment. No late submissions will be accepted. If you work in pairs, list the names and email PIDs of both members at the beginning of the file, and submit your solution under only one PID. No other formats will be graded.

For this assignment, you may (and are encouraged to) work in pairs; if you do so, you must also write your solutions in such a way that it is clear how each member contributed to deriving the solution.

You will submit your answers to the Curator System (www.cs.vt.edu/curator) under the heading OOC04.

For each question below, the quality of your explanation of how you derived the answer will carry as least as much weight as whether you've stated a correct solution. For each problem, apply one of the heuristics discussed in the course notes, and explain how you applied it.

1. [25 points] Solve the following cryptarithm. Explain exactly how you deduced the solution.

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      TELL
      TALE
      TELL
      TALE
      ----
      LATE

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Remember that each letter stands for a different digit (base-10), and that there are no leading zeros.

From the 1s column, we see that $2L + 2E = E + C_0$, or $2L + E = 10 \cdot C_0$, where C_0 is 0 or 1 or 2. Note that 0 is impossible since that would require that L and E both be 0, and 3 is impossible because $2L + E$ can be no larger than 26. This implies E must be even, but nothing else seems to follow immediately.

From the 10s column, we see that $4L + C_0 = T + 10 \cdot C_1$, where C_1 is 0 or 1 or 2 or 3 and C_0 is either 1 or 2.

From the 1000s column, we see that $4T + C_2 = L$, where C_2 is the carry from the 100s column. So, T can be no larger than 2 (or there would be a carry from the 1000s column), and T cannot be 0 since leading 0s are not allowed. So, T is either 1 or 2.

Returning to the observation from the 10s column, if $T = 1$ then $4L + C_0$ must be 1 or 11 or 21 or 31. If $C_0 = 1$ then $4L + C_0$ must be 1 or 21, which implies L would be either 0 (impossible since L is a leading digit in the sum) or 5. If $C_0 = 2$, then $4L + C_0$ cannot be any of 1, 11, 21 or 31. So, if $T = 1$ then L must be 5. On the other hand, if $T = 2$, then $4L + C_0$ must be 2 or 12 or 22 or 32, and a similar analysis shows that L would then have to be 5. So, $L = 5$ in any case.

But then, from the observations about the 1s column, since $2L + E = 10 \cdot C_0$, we must have that $E = 10 \cdot (C_0 - 1)$ and that's only possible if $E = 0$ and $C_0 = 1$.

So, in the 10s column we must have $4 \cdot 5 + 1 = T + 20$, and so $T = 1$.

So now the original problem reduces to:

1055
1A50
1055
1A50

5A10

Now, we see that the carry from the 100s column must be 1, and so in the 100s column we must have that $2 + 2A = A + 10$, and so $A = 8$.

So, the solution is:

1055
1850
1055
1850

5810

In this analysis, I used the notion of a special feature in my analysis of the sum in each column.

2. [25 points] Solve the following classic puzzle from Sam Loyd (1841-1911). Explain your logic clearly.

Here is a puzzle known as the Covent Garden Problem, which appeared in London half a century ago, accompanied by the somewhat surprising assertion that it had mystified the best mathematicians of England:

Mrs. Smith and Mrs. Jones had equal number of apples but Mrs. Jones had larger fruits and was selling hers at the rate of two for a penny, while Mrs. Smith sold three of hers for a penny.

Mrs. Smith was for some reason called away and asked Mrs. Jones to dispose of her stock. Upon accepting the responsibility of disposing her friend's stock, Mrs. Jones mixed them together and sold them of at the rate of five apples for two pence.

When Mrs. Smith returned the next day the apples had all been disposed of, but when they came to divide the proceeds they found that they were just seven pence short, and it is this shortage in the apple or financial market which has disturbed the mathematical equilibrium for such a long period.

Supposing that they divided the money equally, each taking one-half, the problem is to tell just how much money Mrs. Jones lost by the unfortunate partnership?

Note: one thing that may not be clear from the problem statement is that Mrs Smith was called away before either of them had sold any apples.

Here's Loyd's solution:

The mixed apples were sold of at the rate of five apples for two pence. So they must have had a multiple of five i.e. 5, 10, 15, 20, 25, 30, ..., 60, 65, ... etc apples.

(WDM: however, if Mrs Smith was going to sell hers at 3 for a penny, the number of apples she had must have also been a multiple of 3; and similarly the number of apples Mrs Jones had must have also been a multiple of 2; and since they had the same number of apples, that must have been a multiple of both 2 and 3, and also of 5. The smallest number of apples each could have had is therefore 30.)

But the minimum number of apples they could have together is 60; so that 30 would have been of Mrs. Smith's that would fetch her 10 (an integer) pence and the other 30 of Mrs. Jones's that would fetch her 15 (also an integer) pence.

When sold separately it would fetch them $10+15=25$ pence altogether. But when sold together it would fetch them $60 \times 2/5 = 24$ pence i.e. a loss of one ($25-24=1$) pence.

Since they lost 7 pence altogether; they had altogether $60 \times 7 = 420$ apples that fetched them only $420 \times 2/5 = 168$ pence and they shared 84 pence each of them. But Mrs. Jones could sell her $420/2=210$ apples for $210/2=105$ pence so she lost "21 pence".

I applied the notion of a special feature in taking advantage of the fact that the number of apples had to be a multiple of certain divisors, and the fact that each woman had the same number of apples.

3. [25 points] A 32-page newspaper (think of a larger Collegiate Times) consists of 8 full sheets of newsprint. One of these sheets is selected at random, and the four page numbers that appear on it are added together. Can you determine the resulting sum? If so, what is it, and why?

The key is to recognize the relationship among the four page numbers that occur on each sheet. Consider the sheets beginning with the one containing page 1 of the paper:

1st sheet: 1, 2, 31, 32
2nd sheet: 3, 4, 29, 30
3rd sheet: 5, 6, 27, 28

We see that the i -th sheet will have pages numbered $2i-1$, $2i$, $32-2i+1$ and $32-2i+2$.

So, if we add the four page numbers, the variable i cancels out and we obtain the sum 66.

I would say I took advantage of symmetry in this problem, since there is a natural symmetry between the smaller and larger page numbers that occur on each sheet.

4. [25 points] The CSI team has discovered a body in a storage unit whose interior dimensions are 10 feet by 10 feet by 10 feet. Due to the maturity of the corpse, there are 2001 flies inside the storage unit. Prove that, at any moment, there must be a group of three or more flies that would fit inside a sphere with radius 1 foot.

The storage unit can be viewed as a block of $10 \times 10 \times 10$ 1-foot cubes. So, there are 1000 cubical spaces within the storage unit. If each of them contained more than 2 flies then there could be no more than 2000 flies within the storage unit. Therefore, there must be a cube that contains three or more flies.

(This is just an application of the general form of the Pigeonhole Principle.)

Now, the diagonal of a 1-foot cube is $\sqrt{3}$ feet long, which is smaller than 2 feet. Therefore, a 1-foot cube will fit within a sphere of radius 1 foot (or diameter 2 feet).

So, there is a sphere of radius 1 foot that contains three or more flies.

In this case, I obviously applied the Pigeonhole Principle.