Some Problems

Even fairly good students, when they have obtained the solution of the problem and written down neatly the argument, shut their books and look for something else.

Doing so, they miss an important and instructive phase of the work. …

A good teacher should understand and impress on his students the view that no problem whatever is completely exhausted.

George Polya

Special Features

Common metaphors for problem solving:

- Moving forward
- Making progress

When you are stuck, how do you move forward?

Hints can help… if you can get one

How do you "give yourself" a hint?

Look for special features in the problem.

Searching the Problem Space

```
    E A T
+ T H A T
---------
A P P L E
```

Standard rules:

- Letters consistently map to numbers
- No leading zero (common use of numbers)
- The numbers must work to add up correctly

What is special here, to get us started?

Another Example

```
   L E T S
+  W A V E
-----------
L A T E R
```

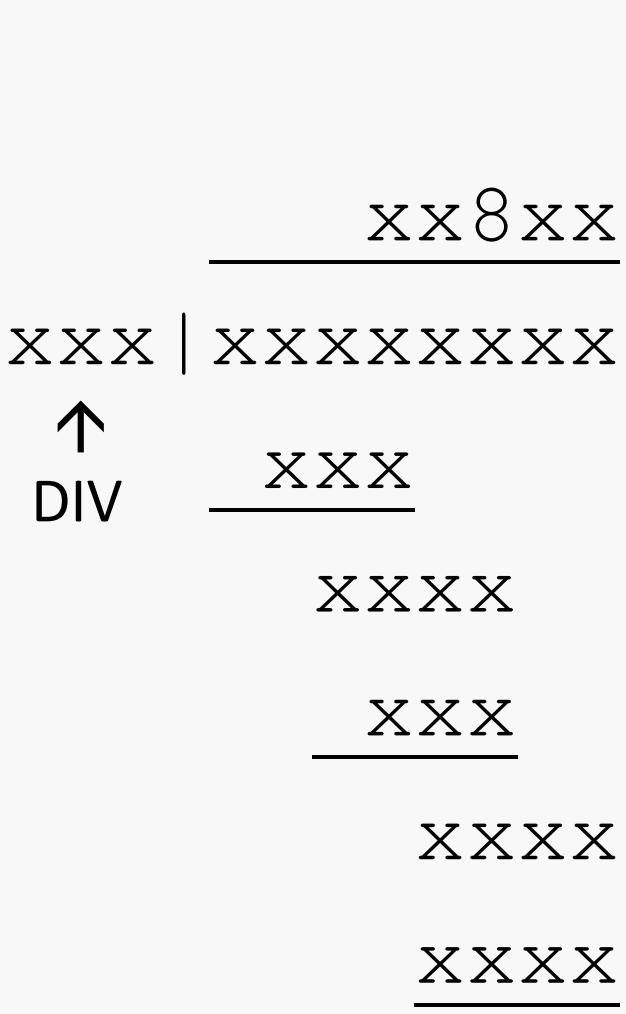
Another Crypt-Arithmetic Problem

```
   D O N A L D
+  G E R A L D
-------------
   R O B E R T
```

Division Problem

```
      xx8xx
   ---------
xxx | xxxxxxxxx
      xxx
     ----
      xxxx
      xxx
     ----
       xxxx
       xxxx
      ----
```

Division Problem: Overview



Substitute a digit between 0-9 for each X. Initial digits are never 0.

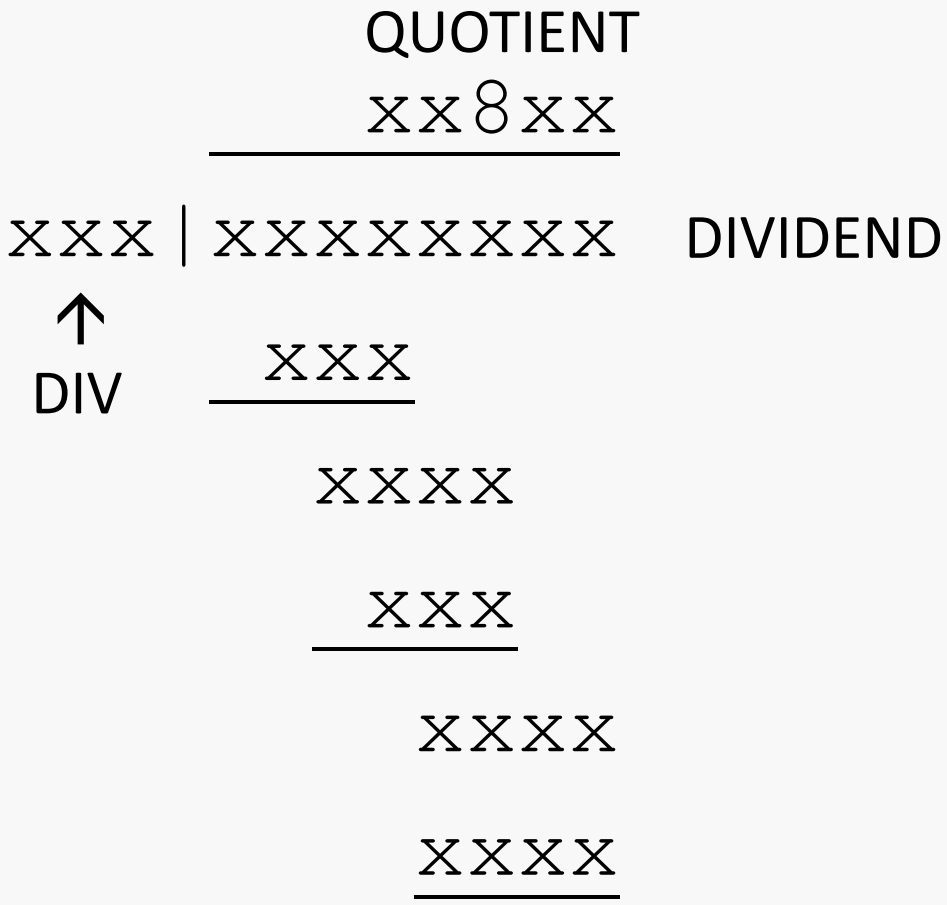
Look for special features.

← PROD 1

← PROD 2

← PROD 3

Division Problem: Getting Started



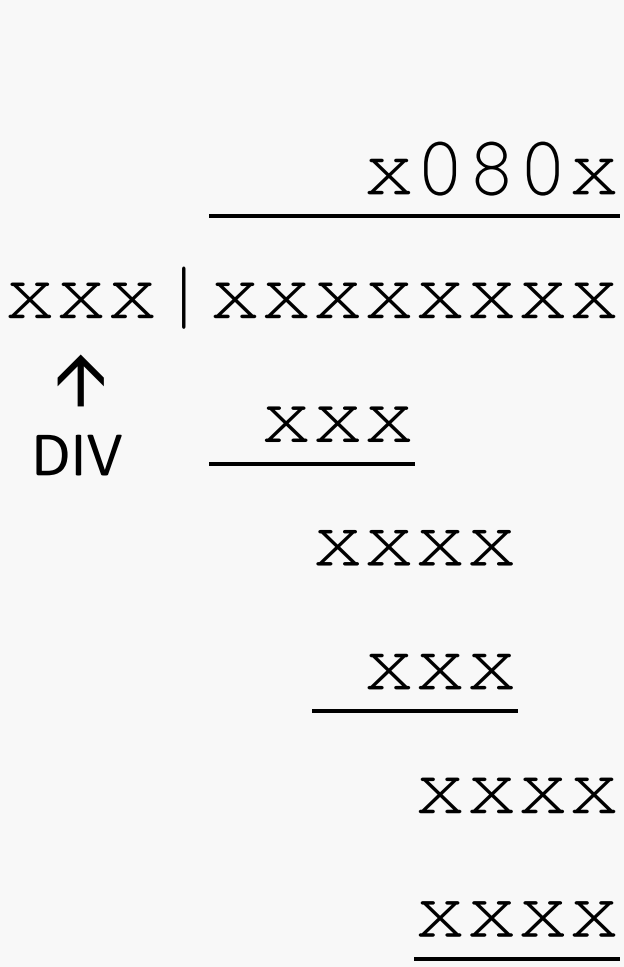
Special features:

PROD 1 not placed under the first 3 digits?

We "skip" a column between the first digit of the quotient and the 8; that means the intervening digit must be a 0.

Likewise, the next-to-last digit in the quotient must be a 0.

Division Problem: Getting Started



← PROD 1

← PROD 2

← PROD 3

Special features:

PROD 2 result of known multiplier: 8

Multiplying DIV by 8 yields a 3 digit number.

Thus DIV must be a small number in the range 100-124 (since 125*8 = 1000).

So PROD 2 is a number between 800-992.

Division Problem: Special Features


```
PROD          PROD
2             2

xxxx          xxxx
-8xx          -9xx
----          ----
 xx            xx
```

```
 10xx
-9xx
----
 xx
```

Special features:

PROD 2 is a number between 800-992.

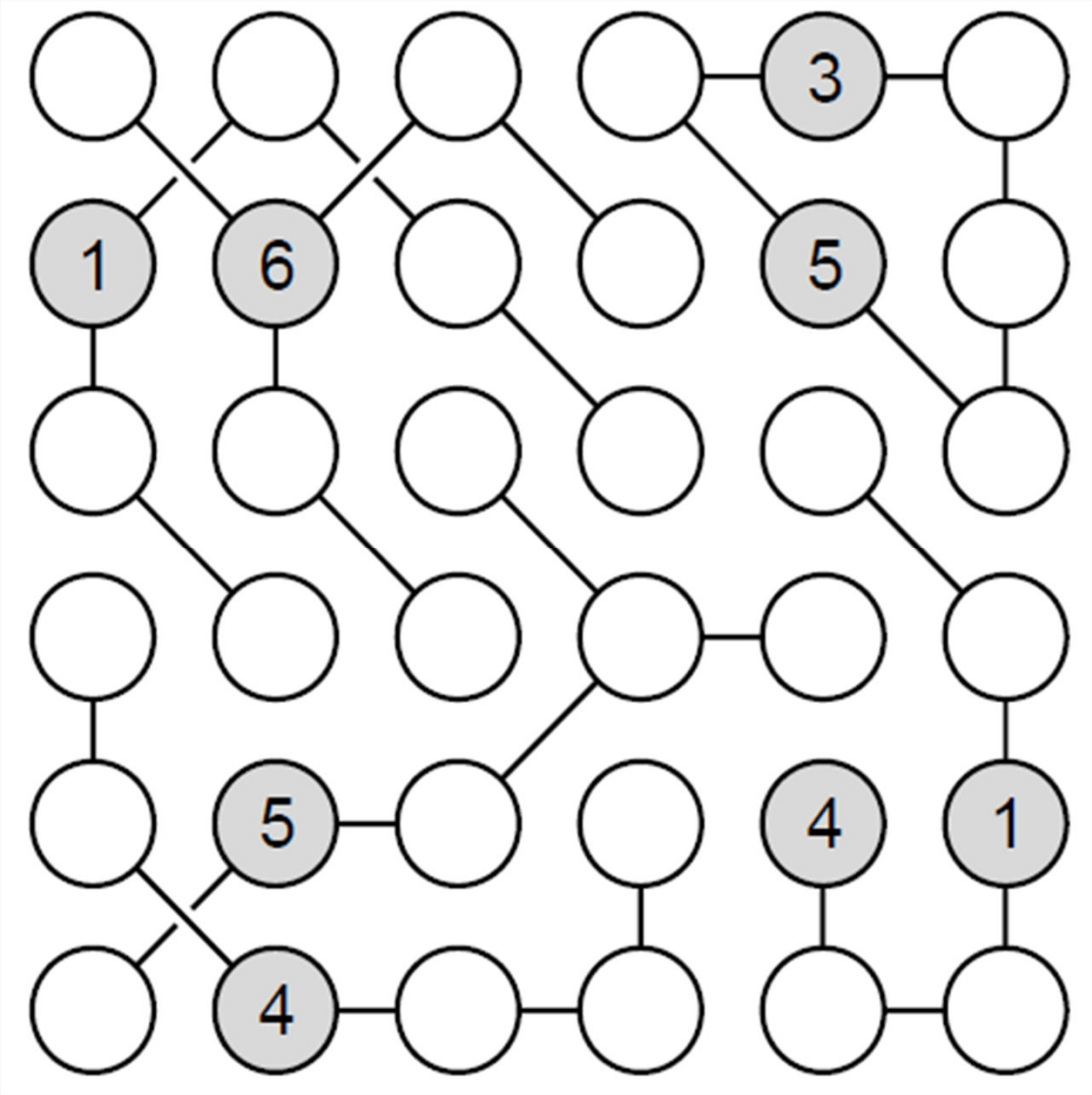
It is subtracted from a 4 place number but yields a 2 place result. The only combination for which this can hold is when a 1 is borrowed to the second column to cancel a 9.

Therefore DIV*8 > 900 so 113 $\leq$ DIV $\leq$ 124.

Sudoku Puzzles

		1				8	9	
	2	7			9		5	
		4		8	2			
	6		9	2		1	4	
				5				
	9	8		6	1		3	
			2	1		4		
	1		7			3	6	
	7	9				2		

Generalizations



Strimko

Special Cases

A man leaves his camp by traveling due north for 1 mile. He then makes a right turn (90 degrees) and travels due east for 1 mile. He makes another right turn and travels dues south for 1 mile and finds himself precisely at the point he departed from, that is, back at his campsite. Where is the campsite located (or where on earth could such a sequence of events take place)?

Go to Extremes

Manipulate the problem space

Look at extreme limits of the problem space.

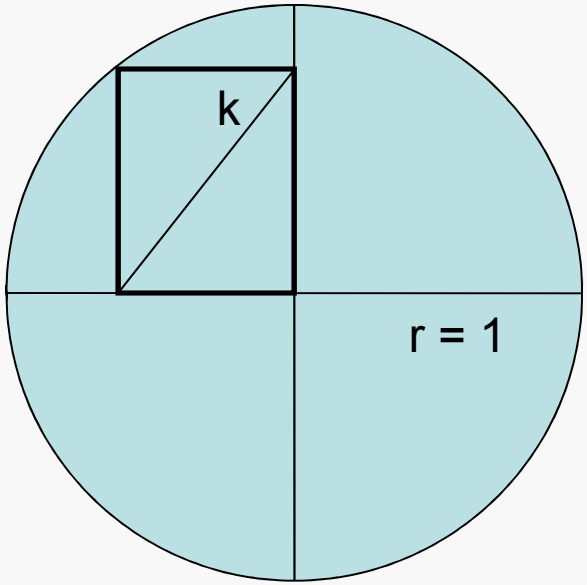
Example

Two flagpoles are standing, each 100 feet tall. A 150-foot rope is strung from the top of one of the flagpoles to the top of the other and hangs freely between them. The lowest point of the rope is 25 feet above the ground. How far apart are the two flagpoles?

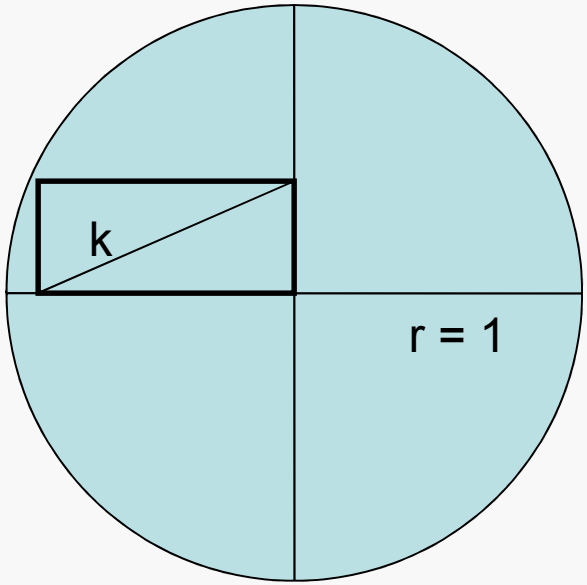
Hint: Start by drawing pictures.

Example

What is the length of k?



Important fact: k remains the same no matter what rectangle is inscribed:

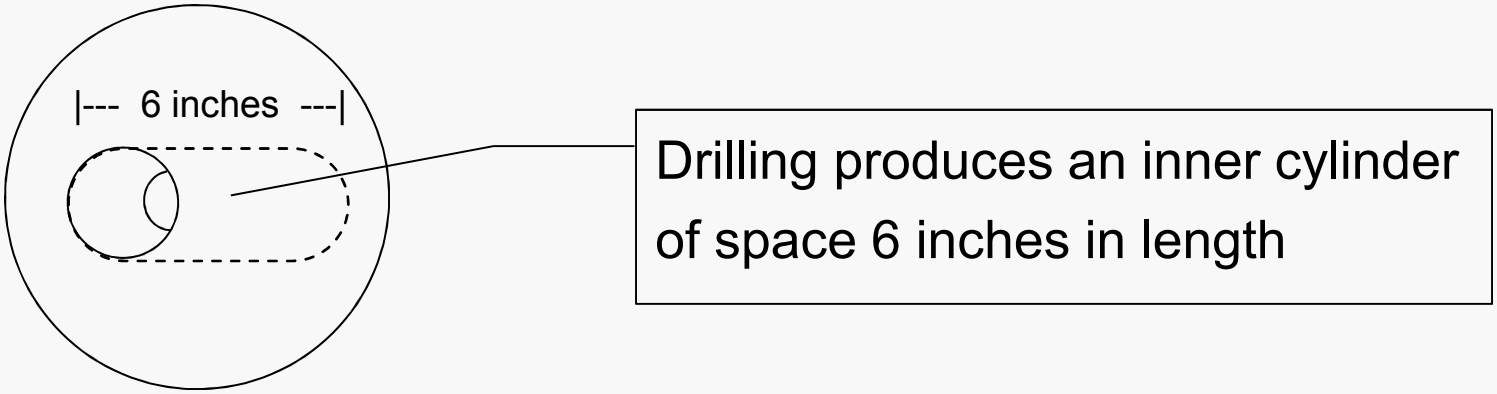


Example

You have a large, solid sphere of gold. A cylinder of space is "bored" through the center of this sphere, producing a ring. The length of that cylindrical line is 6 inches. You want to know how much gold you have left in the ring. Specifically, what is the volume of the ring?

Note: for any sphere,

$$V = \pi D^3/6.$$



Simplify the Problem…

Take a number of several digits (say 7 or 8 digits). Reverse it and calculate the difference. Now if you tell me all but one of the digits in the answer (in any order), I can tell you the missing digit.

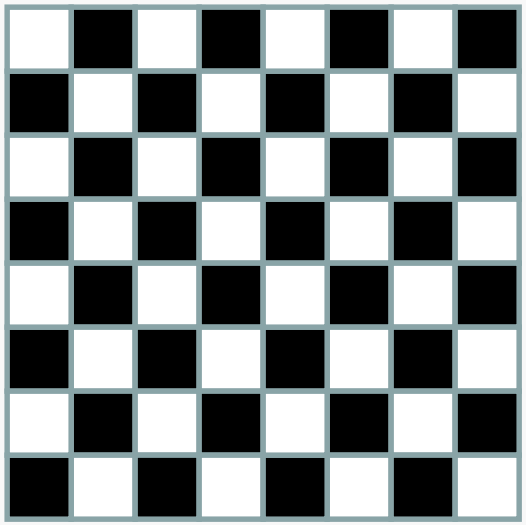
How can you go about figuring out the method?

- You can try some examples and look for a pattern.
- But if you do it on big numbers, it will be hard to figure out.

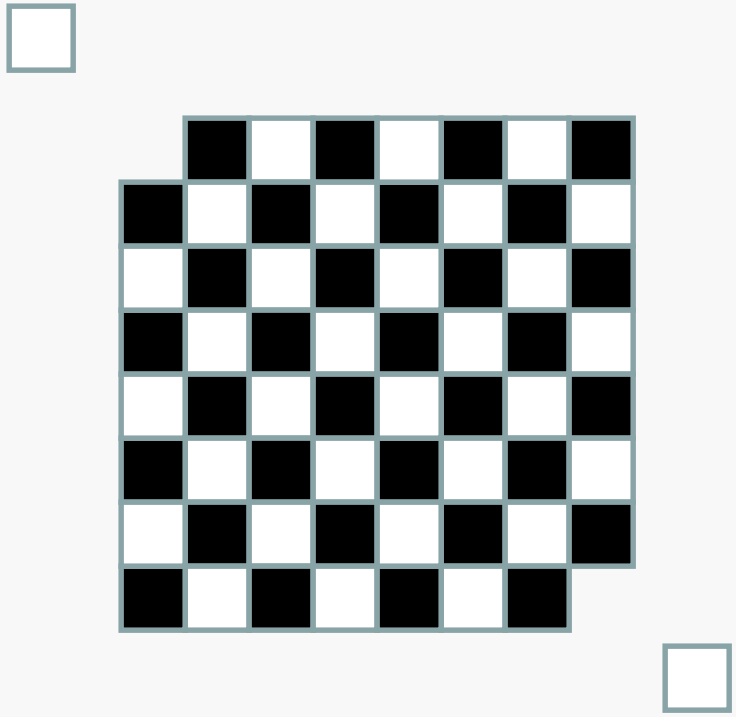
Chessboard Problem

A domino covers two squares of a chessboard.

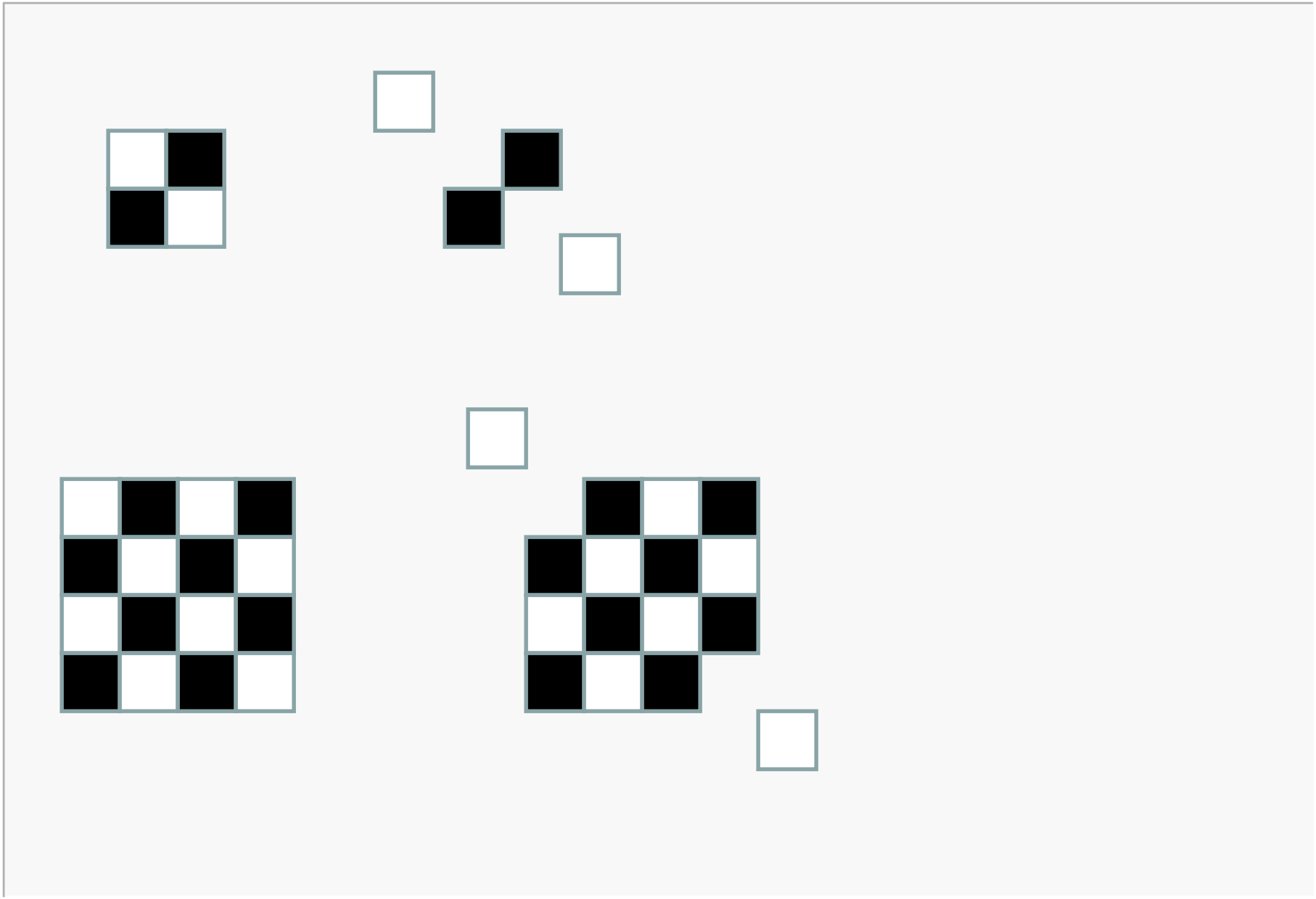
1. Can a chessboard be covered by dominos without any dominos sticking out?
2. Now, cut off the upper-left and lower-right corners of the chessboard. Can it still be covered by dominos completely?



Visualization



Simplify



Coins 1

You have 24 coins that look alike. With the exception of one counterfeit, they are all made of gold and weigh exactly the same. The "bad" coin is either heavier or lighter than the others, you do not know which. You also have available an old-fashioned balance scale.

In the worst case, what is the minimum number of weighings you must make in order to locate the bad coin?

Coins 2

You are given 10 stacks of what should be 10 gold pieces each. Each gold piece weighs two ounces. Unfortunately, one stack contains 10 counterfeits, each coin weighing only one ounce. You have a kitchen-type scale that reads out the weight of what is put on it.

The problem: determine the counterfeit stack with a single weighing.