Logic is the anatomy of thought.

John Locke

When introduced at the wrong time or place, good logic may be the worst enemy of good teaching.

George Polya

A *proposition* is a statement that is true or false:

Di-hydrogen monoxide is liquid at a temperature of 0° Fahrenheit.

The Gregorian calendar uses twelve months.

Every even integer greater than 2 can be expressed as the sum of two prime integers.

Not all declarative statements are propositions:

Fried chicken livers taste wonderful.

This sentence is false.

Propositional logic is concerned with the "algebra" of propositions.

We may form new propositions from old ones by performing certain "algebraic" operations. Consider the following two rather dull propositions:

> 51 is a prime integer.

> The set {3, 5, 7, 11} contains only prime integers.

Then the following are also propositions:

> 51 is <u>not</u> a prime integer.

> 51 is a prime integer <u>or</u> the set {3, 5, 7, 11} contains only prime integers.

> 51 is a prime integer <u>and</u> the set {3, 5, 7, 11} contains only prime integers.

And so are the following:

If 51 is a prime integer, <u>then</u> the set {3, 5, 7, 11} contains only prime integers.

If the set {3, 5, 7, 11} contains only prime integers, <u>then</u> 51 is a prime integer.

If the set {3, 5, 7, 51} contains only prime integers, <u>then</u> 51 is a prime integer.

Suppose that P and Q are propositions, then so are the following:

not P	*negation*	$\neg P$
P and Q	*conjunction*	$P \wedge Q$
P or Q	*disjunction*	$P \vee Q$
if P then Q	*implication*	$P \rightarrow Q$

Suppose that P and Q are propositions, then:

not P	is true if and only if P is false
P and Q	is true if and only if P is true and Q is true
P or Q	is true if and only if P is true, or Q is true, or both are true
if P then Q	is true if and only if P is false or Q is true

P	not P
T	F
F	T

P	Q	P and Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P or Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	if P then Q
T	T	T
T	F	F
F	T	T
F	F	T

Intro Problem Solving in Computer Science

Notes:

- "or" is *inclusive*; that is it's true as long as at least one "side" is true

- "if… then" is false only in the case that the *antecedent* is true and the *consequent* is false

- "if… then" has absolutely nothing to do with causation

- If you're confused by the truth table for "if…then", consider such statements as being a contract. If you say

    If today is Wednesday then I will give you a dollar.

precisely what are your obligations if today is not Wednesday?

A propositional expression is a *tautology* if the expression is always true, no matter what truth values its components may have.

Example: P or not P.

Now, if P is true then not P must be false, and therefore the expression is true (by definition of "or").

On the other hand, if P is false then not P must be true, and again the expression is true.

This may also be expressed by the construction of a *truth table* that considers all cases:

P	P or not P
T	T
F	T

An *inference rule* is rule that allows us to infer a conclusion from given conditions (the *premise*).

Inference rules take the following general form:

        Premise P1.
        Premise P2.
        . . .
        Premise PN.

        Therefore, Conclusion.

An inference rule must have the property that the conclusion must be true whenever all the premises are true.

Equivalently, the following expression must be a tautology:

                (P1 and P2 and … and PN) → Conclusion

We may form the truth table for the corresponding expression:

P or Q.
not P.
---------------------   *Modus tollendo ponens*
therefore, Q.

P	Q	P or Q	not P	(P or Q) and (not P)	(P or Q) and (not P) → Q
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

Note that we actually only needed to construct the truth table for the case in which the antecedent of the implication was true (third row).

Why?

The last observation leads to a somewhat simpler verbal analysis:

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P or Q.
not P.
----------------------          Modus tollendo ponens
therefore, Q.
```

Assume the premises are both true. (This is the only case in which the argument form could possibly be false.)

Then from the definition of "not", since "not P" is true, P must be false.

But if "P or Q" is true, by the definition of "or", P and Q cannot both be false.

Hence, Q must be true.

And so, if both premises are true then the conclusion must also be true.

Here's one of the most basic inference rules (we assume that P and Q represent specific propositions):

> if P then Q.
> P.
> ----------------   *Affirming the antecedent (Modus ponens)*
> therefore, Q.

For example, if are given that:

> If Cirrus is a cat, then Cirrus is carnivorous.
> Cirrus is a cat.

Then we may infer:

> Cirrus is carnivorous.

An argument is *valid* if the conclusion is true whenever the premises are true.

An argument is *sound* if it is valid and the premises are, in fact, true.

The following argument is valid, but not sound:

Premises:
If January has 35 days then January contains 5 Mondays.
January has 35 days.
Conclusion:
January contains 5 Mondays.

Propositional logic is largely concerned with the issue of how to form valid arguments. The soundness of an argument generally depends upon external (domain-specific) knowledge.

Note that whether an argument is valid has nothing whatsoever to do with whether its premises are true or its conclusion is true:

Premises:

  If Socrates was bipedal, then Socrates was a philosopher.
  Socrates was bipedal.

Conclusion (via Modus ponens or Affirming the antecedent):

  Socrates was a philosopher.

Premises:

  If Grace Hopper was Australian, then she was a computer scientist.
  Grace Hopper was Australian.

Conclusion:

  Grace Hopper was a computer scientist.

Premises:

  If Socrates was from Rome, then Socrates spoke Latin.
  Socrates was from Rome.

Conclusion:

  Socrates spoke Latin.

Again, assume that P and Q represent specific propositions:

if P then Q.
not Q.
----
therefore, not P.

Denying the consequent (Modus tollens)

P or Q.
not P.
----
therefore, Q.

Disjunctive syllogism (Modus tollendo ponens)

not not P.
----
therefore, P.

Double negation

P and Q.
----
therefore, P.

Conjunction elimination

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if P then Q.
if not P then Q.
----------------------
therefore, Q.
```

Assume the premises are both true. Also, note that P must be either true or false (although we have no idea which is the case).

Now, if P is true, then Q is true from the first premise via *modus ponens*.

And, if P is false, then not P is true, and therefore Q is true from the second premise via *modus ponens*.

Therefore, no matter whether P is true or P is false, Q must be true.

Why do we care?

If we want to derive results from a collection of alleged facts (premises) that we possess, we must understand what will guarantee that out results do, in fact, follow from the premises we are given.

Natural languages, especially English, promote using grammatical constructs that lend themselves to obscuring the underling logic, or lack thereof.

And, in any language, there may be implicit assumptions that are required to justify the stated conclusions. If those assumptions are false, then the argument is unsound, even if it is valid (and appears to be sensible when read without a full understanding of the true logic).

Consider the exchange quoted on the following slide…

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This was taken from an episode of a TV show:

Bob Murch, spirit board collector:
"There's been thousands of years of accounts of ghosts and hauntings, and if those are true, you know, surely a spirit board can work."

Penn Jillette:
"So, if those aren't true, a spirit board can't work? Cool!"

Source: Penn & Teller, "Ouija Boards/Near Death Experiences", *B.S.!*

Is there a flaw in Penn Jillette's logic?

How can you be sure?

How can you convince anyone of your conclusion?

This was taken from the Roanoke Times for January 22, 2011:

Lincoln said the war was about the Union

Re: "Slavery was central to Civil War," Jan. 10 commentary:

When I read this essay, it became apparent the author is either a Yankee or someone boasting about his background in history. Maybe both, for him to go out of the way to put down the South as he did.

Just one question: If slavery was so central to causing the war to prevent Southern independence, why did President Lincoln fire Gen. John Fremont for declaring slaves would be set free after the Battle of Wilson's Creek in August 1861 and make the statement: "This war is being fought for a great national idea, the Union, and the general should not have dragged the Negro into it"?

Is the writer's logic valid? Is it sound?

Are there hidden assumptions (premises)?

This was taken from Aristotle's *Politics*:

> Every state is a community of some kind, and every community is established with a view to some good; for mankind always act in order to obtain that which they think good.
>
> But, if all communities aim at some good, the state or political community, which is the highest of all, and which embraces all the rest, aims at good in a greater degree than any other, and at the highest good.

Is the writer's logic valid? Is it sound?

Are there hidden assumptions (premises)?

This problem was taken from the *Puzzlers' Paradise* website:

Zookeeper George was in charge of feeding all of the animals in the morning.   He had a regular schedule that he followed every day. Can you figure it out from the clues?

P1:   Feedings begin at 6:30 am.
P2:   A feeding takes 15 minutes.
P3:   The last feeding begins no later than 7:30 am.
P4:   The giraffes were fed before the zebras but after the monkeys.
P5:   The bears were fed 15 minutes after the monkeys.
P6:   The lions were fed after the zebras.

A carefully-reasoned analysis will provide a sequence of well-explained inferences, reaching a specific conclusion.

P4: The giraffes were fed before the zebras but after the monkeys.

By Conjunction Elimination, P4 implies:
  I1: Monkeys are fed before Giraffes.
  I2: Giraffes are fed before Zebras.

P1: Feedings begin at 6:30 am.
P2: A feeding takes 15 minutes.

P2 and I1 imply
  I3: Giraffes are fed at least 15 minutes after Monkeys.

I3 and P1 imply
  I4: Giraffes are fed no earlier than 6:45 am.

P4: The giraffes were fed before the zebras but after the monkeys.

By Conjunction Elimination, P4 implies:
**   I1: Monkeys are fed before Giraffes.**
**   I2: Giraffes are fed before Zebras.**

P1: Feedings begin at 6:30 am.
P2: A feeding takes 15 minutes.

P2 and I2 imply
**   I5: Zebras are fed at least 15 minutes after Giraffes.**

I5 and I4 (Giraffes are fed no earlier than 6:45 am.) imply
**   I6: Zebras are fed no earlier than 7:00 am.**

P5: The bears were fed 15 minutes after the monkeys.

    P1, P2, P5 and I3 imply
        I7: Giraffes are fed at least 30 minutes after Monkeys.

    I7 and P1 imply
        I8: Giraffes are fed no earlier than 7:00 am.

I8 and I2 imply
        I9: Zebras are fed no earlier than 7:15 am.

P2: A feeding takes 15 minutes.
P6: The lions were fed after the zebras.
I9:   Zebras are fed no earlier than 7:15 am.

> **P2, P6 and I6 imply**
>> **I10: Lions are fed no earlier than 7:30 am.**
>
> **P3 and I10 imply**
>> **I11: Lions are fed at 7:30 am.**

I3 – I7 imply a linear ordering:
>> **I12: Monkeys precede Bears, which precede Giraffes,**
>> **which precede Zebras, which precede Lions.**

P1 - P3 imply
> **I13: There are five possible feeding times:**
> > **6:30, 6:45, 7:00, 7:15 and 7:30.**

So I12 and I13 imply that
> **George feeds Monkeys first at 6:30,**
> > **then Bears at 6:45,**
> > **then Giraffes at 7:00,**
> > **then Zebras at 7:15,**
> > **and finally Lions at 7:30.**