READ THIS NOW!

- Print your name in the space provided below.
- There are 4 short-answer questions, priced as marked. The maximum score is 100.
- The grading of each question will take into account whether you obtained a correct solution and how well you presented your analysis and justified your logic. In most cases, as much weight will be given to the presentation and explanation of your analysis as to whether the solution is fully correct. Legibility will be strongly considered in the grading. You may use scratch paper to work out your solution before finalizing it on the exam.
- Externalize! Whether it's a drawing, a table, an equation or something else, externalize! And make the externalization explicit in your answer! Label things for clarity!
- Look for opportunities to apply the various techniques that have been discussed in class.
- You may use the supplied extra paper for scratch work. Write your name on any scratch work sheets you use and turn those in with your exam.
- All final answers must be written on the test form itself.
- When you have finished, sign the pledge at the bottom of this page and turn in the test.
- This is a closed-book, closed-notes examination.
- No laptops, calculators, cell phones or other electronic devices may be used during this examination.
- Until solutions are posted, you may not discuss this examination with any student who has not taken it.
- Failure to adhere to any of these restrictions is an Honor Code violation.

Name (Last, First) Solution

printed

Pledge: On my honor, I have neither given nor received unauthorized aid on this examination.

_____

signed

1. [30 points] The cryptarithmetic problem shown below has multiple solutions. However, the problem has three special features that allow you to quickly deduce what the values of three of the unknowns must be. Describe each of these special features, and the conclusion to which it leads. (You do not have to determine a complete solution.)

Recall that each letter stands (consistently) for a different base-10 digit, and that there are no leading zeros.

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 CAT
 RAT
 BAT
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SCAT
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1: In the 10^0 column, we have that T + T + T equals either T or T + 10. Therefore, T must be either 0 or 5.

2: If T = 0, then in the 10^1 column we have A + A + A equals either A or A + 10; so if T = 0 then A must be either 0 (but that's impossible since T = 0 already) or A must be 5.

If T = 5, then in the same column we have A + A + A + 1 equals either A or A + 10; but, neither of those is possible since each would require A to not be a digit.

So, we know that T = 0 and that A = 5.

3: The S in the 10^3 column must result from a carry out of the 10^2 column, and so S is either 1 or 2.

But, a closer look shows that in the 10^2 column we must get the sum C + R + B + 1, since we now know there's a carry from the 10^1 column, and that sum must equal either C + 10 or C + 20 (since we must have a carry out from the 10^2 column).

So, considering the cases, we have R + B + 1 equaling either 10 or 20; but the latter is impossible. So, the carry out from the 10^2 column must be 1.

So, we know that S = 1.

2. [20 points] Prove that among any group of five people, there must be at least two people who have the same number of acquaintances within the group. (Acquaintance is a symmetric relationship.)

Let's call the people P1 through P5, and let's call the number of acquaintances Pk has within the group A(Pk).

Then we seem to have 5 possible values for A(Pk): 0, 1, 2, 3, 4, 5.

But, if there's a k such that A(Pk) is 0, then there cannot be a j such that A(Pj) is 4; and vice versa.

So, there are really only 4 possible values for A(P1), A(P2), A(P3), A(P4) and A(P5).

By the Pigeonhole Principle, there must be two people, say Pk and Pj, such that A(Pk) equals A(Pj).

3. [20 points] A machine shop produces brass couplings, each of which must be marked with an integer serial number before being shipped. Serial numbers are assigned as consecutive integers, starting at 1. A serial number is marked onto a coupling by a worker who has ten steel dies that she can use to stamp a single digit onto the coupling. For example, to mark a coupling with the serial number 732, the worker would use the dies for the digits '7', '3' and '2', striking each with a hammer one time to emboss the corresponding digit on the coupling. When the worker finishes marking a shipment of couplings, she realizes that she has used her hammer exactly 399 times. What was the serial number that she marked on the last coupling? Externalize, and justify your conclusion.

Stamping 1-digit serial numbers on the first nine couplings ("1" through "9") would require 9 blows.

Stamping 2-digit serial numbers on the next ninety couplings ("10" through "99") would require 180 blows.

That gives us a total of 189 blows for 1- and 2-digit serial numbers; the rest must be for 3-digit serial numbers, since only 210 more blows occurred.

210 blows would account for 70 3-digit serial numbers, since each would require 3 blows.

So she must have stamped a total of 169 serial numbers, and the last one was "169".

4. [30 points] Molly traded in her old car for a one-year-old, high-end car that had a special keyless entry system, which allows her to use her key to unlock the doors and start the engine from a short distance. Everything about the car worked perfectly, with one exception:

When Molly would park the car, sometimes she would come out to find its motor running. In each instance, she had turned the motor off and taken the key out before leaving the car. This problem most often occurred at the Starbucks on Washtenaw Avenue, a major thoroughfare, but seldom at the Starbucks on Liberty Street, which was a more secluded street. It also occurred a couple of times at Dunkin Donuts on Stadium Street.

Even though the problem mainly occurred on Washtenaw, Molly did not want to stop going there because she enjoyed chatting with the police officers who frequented that location. The Liberty Street Starbucks seemed to have only an occasional police officer stop by. The problem arose during the midmorning hours but not over lunch or dinner time.

Carry out a K.T. problem analysis to determine the reason Molly's car is starting spontaneously. Recall that K.T. problem analysis is performed using a table like the one shown below. Of course, you must create your own, larger table. Don't be shy about using space; the next page is left blank intentionally.

	IS	IS NOT	Distinction	Probable cause
What?	spontaneous starting	other electro-mechanical issue; regular starting mechanism	related to keyless ignition system	short in system? spurious initiation signal?
Where?	Starbucks and Dunkin Donuts shops mostly at Starbucks on Washtenaw	other locations seldom at Starbucks on Liberty seldom (?) at Dunkin Donuts on Stadium	very specific locale major street vs secluded street? frequent police presence vs less ??	environmental effect? ?? police officers or vehicles effect system?
When?	midmorning using new car	lunch, dinner using old car	snack time? police taking morning break? keyless ignition system	??
Extent?	occurs sometimes locations frequented by police	occurring all the time locations where no police are present	intermittent cause police activity	underlying cause is not continuous police-related signals?

So, the common factors seem to point to police electronic devices that must use the same frequency as the keyless ignition system.