CS 2104 Homework Assignment 5. Group work

THE PROBLEM (60 points. ”Game theory: real life”.)

An unusual game show was run sometime in the 60s. On the stage there were 3 identical closed doors. The show-host would tell the audience that there were $10,000 behind one of the doors (only the host knew which one). A lucky volunteer was selected from the audience and offered to make his pick: if he guessed the door right, the $$ was his. Once the volunteer realized that the doors were identical, and there were no hints from anybody, he would select one based on whatever random thought process (there were no cell phones at that time to call a local psychic). However, right before he would open the door to check if the prize was behind it, the show host would approach the guy with an unusual offer. He would open one of the remaining doors to show that there was *no* prize behind it (remember that the show-host knew where the money was, so he could always open the one “empty” door). The show-host would then offer the volunteer to *switch* to the one remaining closed doors. If the volunteer accepted, he could not switch back and had to open the new door. If he rejected, he had to open the one he chose in the first place. Once he opened a door, the game was over.

Question: What is the optimal game strategy for the volunteer, that is should he switch or stay by the first door to maximize the probability of winning the prize?

Approach: You will approach the problem from two angles: empirical and computational.

(a) Empirical: Use probability theory to estimate the probability of the win in both strategies: “switch” and ”stay”.

(b) Computational: Simulate the game process. By doing a “random” walk in the event space you can simulate different outcomes. Use random numbers to “assign” the volunteer randomly to one of the doors (without loss of generality you can assume the prize is always placed behind door # 1). This is the first “move”. The second move corresponds to the “switch” strategy. It is not random. By “playing” the game $N \gg 1$ times you can estimate the probabilities of the win in each strategy. Remember that probability of an event equals the ratio of the number of outcomes where the event occurs to the total number possible outcomes N, in the limit of large N. Make sure you present a plotis of the appropriate probabilities vs. N to show convergence as N increases.

Strategy The group will have to convene at least twice. At the first meeting, at least a week before the deadline, you brainstorm. Try to come up with a solution in (a). Decide what is needed for (b). Delegate the programming task. *In your final submission, describe briefly the outcomes of this first meeting.* You will then need to meet a couple of days before the deadline to discuss the whole thing. It is absolutely vital that both (a) and (b) lead to the same results and the same estimates of the win probability for each strategy. Do not leave it until the final hours before the submission: surprises are likely! Make sure you contact your GTAs or the teacher if something is unclear to you in the problem set-up.

Submission Strictly follow the “General Assignment Guidelines” (Group assignment) on the course web-site. Each partnership submits one document, with roles of each partner clearly indicated.