Some Problems

Even fairly good students, when they have obtained the solution of the problem and written down neatly the argument, shut their books and look for something else.

Doing so, they miss an important and instructive phase of the work. …

A good teacher should understand and impress on his students the view that no problem whatever is completely exhausted.

George Polya

Common metaphors for problem solving:
    Moving forward
    Making progress

When you are stuck, how do you move forward?

Hints can help… if you can get one

How do you "give yourself" a hint?

Look for special features in the problem.

```
        E  A  T
     +  T  H  A  T
     _____
        A  P  P  L  E
```

Standard rules:

   Letters consistently map to numbers

   No leading zero (common use of numbers)

   The numbers must work to add up correctly

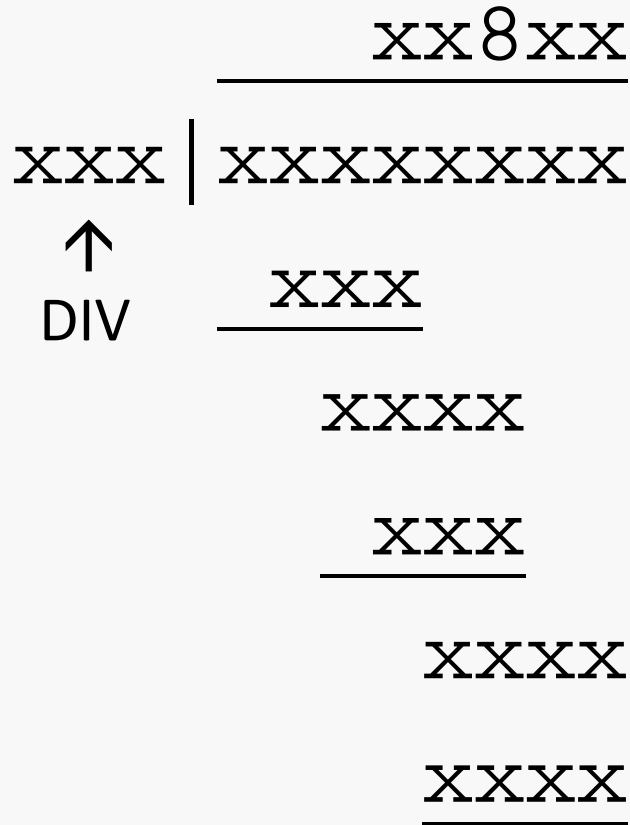What is special here, to get us started?

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```
      L E T S
    + W A V E
    _____

      L A T E R
```

```
      D O N A L D

    + G E R A L D
    _____

      R O B E R T
```

Division Problem

$$\begin{array}{r} \mathrm{xx8xx} \\ \hline \end{array}$$

xxx | xxxxxxxx

   xxx

    xxxx

    xxx

     xxxx

     xxxx

Substitute a digit between 0-9 for each X. Initial digits are never 0.

$$\begin{array}{r} \underline{\text{xx8xx}} \\ \text{xxx}\,\big|\,\text{xxxxxxxx} \end{array}$$

↑
DIV

$\underline{\text{xxx}}$ ← PROD 1

xxxx

$\underline{\text{xxx}}$ ← PROD 2

xxxx

$\underline{\text{xxxx}}$ ← PROD 3

Look for special features.

```
                    QUOTIENT
                    xx8xx
              _____
       xxx | xxxxxxxx      DIVIDEND
        ↑
       DIV     xxx
              _____
              xxxx

              xxx
             _____

               xxxx

               xxxx
              _____
```
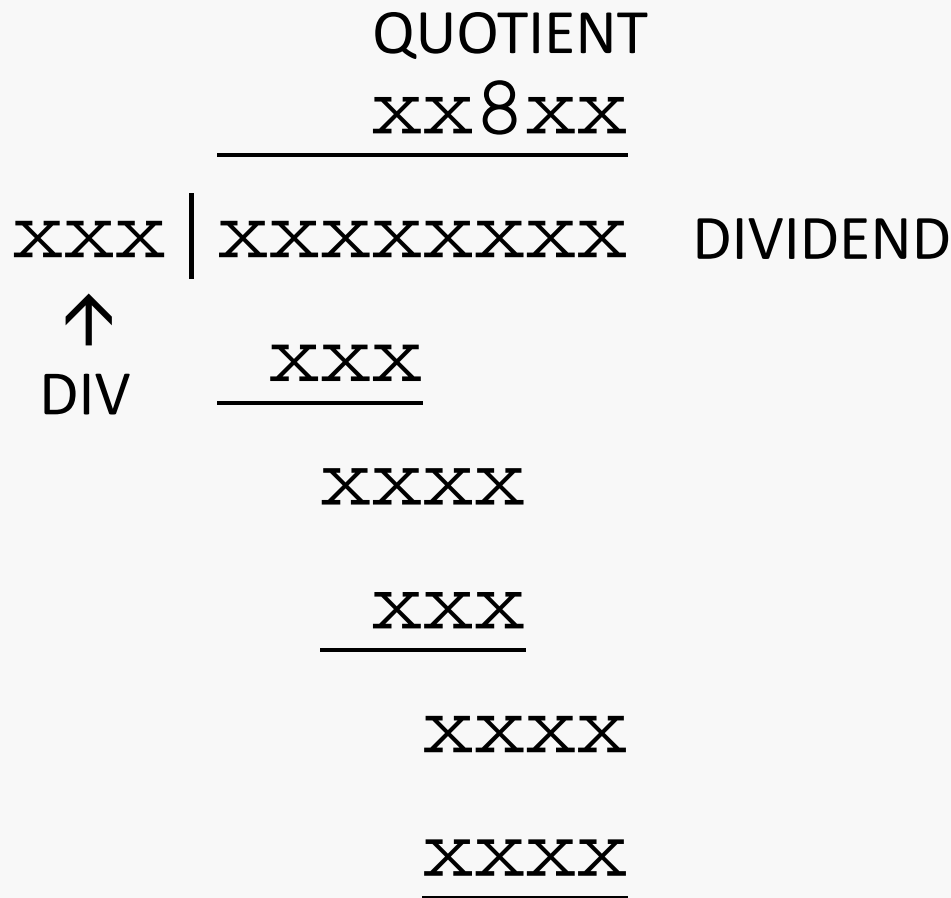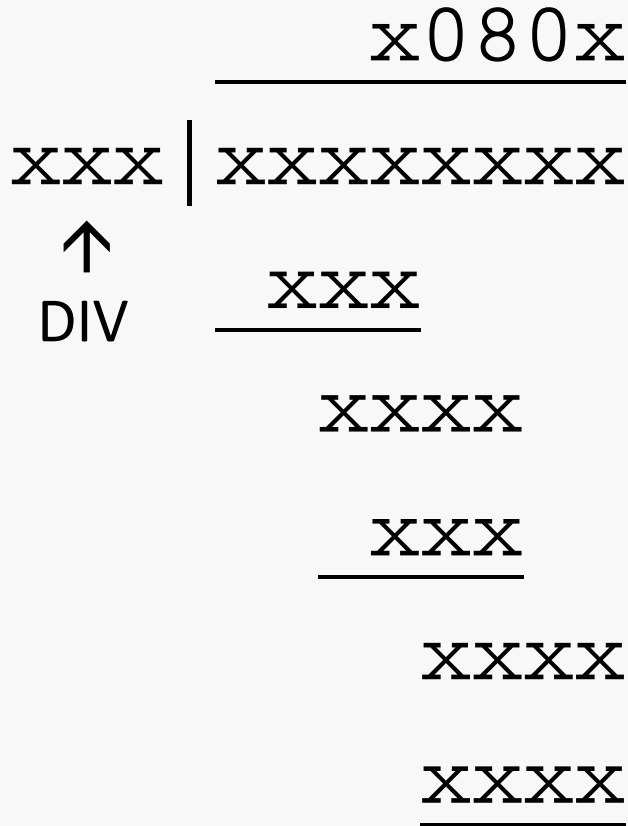
Special features:

PROD 1 not placed under the first 3 digits?

We "skip" a column between the first digit of the quotient and the 8; that means the intervening digit must be a 0.

Likewise, the next-to-last digit in the quotient must be a 0.

Special features:

PROD 2 result of known multiplier: 8

```
            x080x
          _____
xxx | xxxxxxxx
 ↑         xxx              ←  PROD 1
DIV       _____
           xxxx
            xxx             ←  PROD 2
          _____
            xxxx
            xxxx            ←  PROD 3
          _____
```

Multiplying DIV by 8 yields a 3 digit number.

Thus DIV must be a small number in the range 100-124 (since 125*8 = 1000).

So PROD 2 is a number between 800-992.

PROD 2

xxxx

$-8xx$

xx

PROD 2

xxxx

$-9xx$

xx

10xx

$-9xx$

xx

Special features:

PROD 2 is a number between 800-992.

It is subtracted from a 4 place number but yields a 2 place result. The only combination for which this can hold is when a 1 is borrowed to the second column to cancel a 9.

Therefore DIV*8 > 900 so $113 \le$ DIV ≤ 124.

$$\begin{array}{r} \texttt{x080x} \\ \hline \texttt{1xx} \,|\, \texttt{xxxxxxxx} \end{array}$$

↑
DIV

$$\underline{\texttt{xxx}} \quad \leftarrow \quad \textbf{PROD 1}$$

$$\texttt{10xx}$$

$$\underline{\texttt{9xx}} \quad \leftarrow \quad \textbf{PROD 2}$$

$$\texttt{xxxx}$$

$$\underline{\texttt{xxxx}} \leftarrow \quad \textbf{PROD 3}$$

PROD 3is the only product with 4 digits & when subtracted yields 0 so it must be identical to the number it is subtracted from. Which derived from PROD 2.

Multiplying DIV by final quotient digit yields a 4 digit number. Thus the final quotient digit > 8 (which yielded a 3 digit number) therefore it is 9.

Since 113 $\leq$ DIV < 124, PROD 3 must be:

9*113 = 1017 $\leq$ PROD 3 < 1116 = 9*124

```
          x0809
     _____
1xx │ xxxxxxxx
  ↑      xxx        ←  PROD 1
 DIV    _____
        10xx
         9xx       ←  PROD 2
        _____
         1xxx
         1xxx      ←  PROD 3
        _____
```

PROD 2

$10xx$

$-9xx$
_____

$1x$

Special features:

PROD 2 is a number between 800-992.
Only 2 digits must be carried down.

What does the one beneath PROD2 imply?

The X's in column have a difference of one. And a one must be borrowed from column 3.
The only pairs for which this holds are 0-9, 0-8, or 1-9 (in the last 2 cases a 1 must be borrowed into the first column).

Thus PROD 2 is either 99X or 98X. Which implies DIV is either 123 or 124.

Note:
1. Whenever a number is carried down and the result is < DIV we place a 0 in the quotient digit.
2. The PROD 1 subtraction pattern is identical to the PROD 2 subtraction pattern. Thus the first quotient digit must an 8.

$$80809$$

$$12x \overline{\smash{)}10xxxxxx}$$

↑
DIV

$$\underline{9xx} \qquad ← \quad PROD\ 1$$

$$10xx$$

$$\underline{9xx} \qquad ← \quad PROD\ 2$$

$$xxxx$$

$$\underline{xxxx} ← \quad PROD\ 3$$

Division Problem: Completion

```
          80809
124 | 10020316
       992
        1003
         992
         1116
         1116
```

DIV is either 123 or 124 and the quotient must be 80809. By testing each of these possible divisors by the quotients to produce the products one will find that only 124 yields a result that satisfies all the constraints.

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Sudoku Puzzles

		1				8	9	
	2	7			9		5	
		4		8	2			
	6		9	2		1	4	
				5				
	9	8		6	1		3	
			2	1		4		
	1		7			3	6	
	7	9				2		

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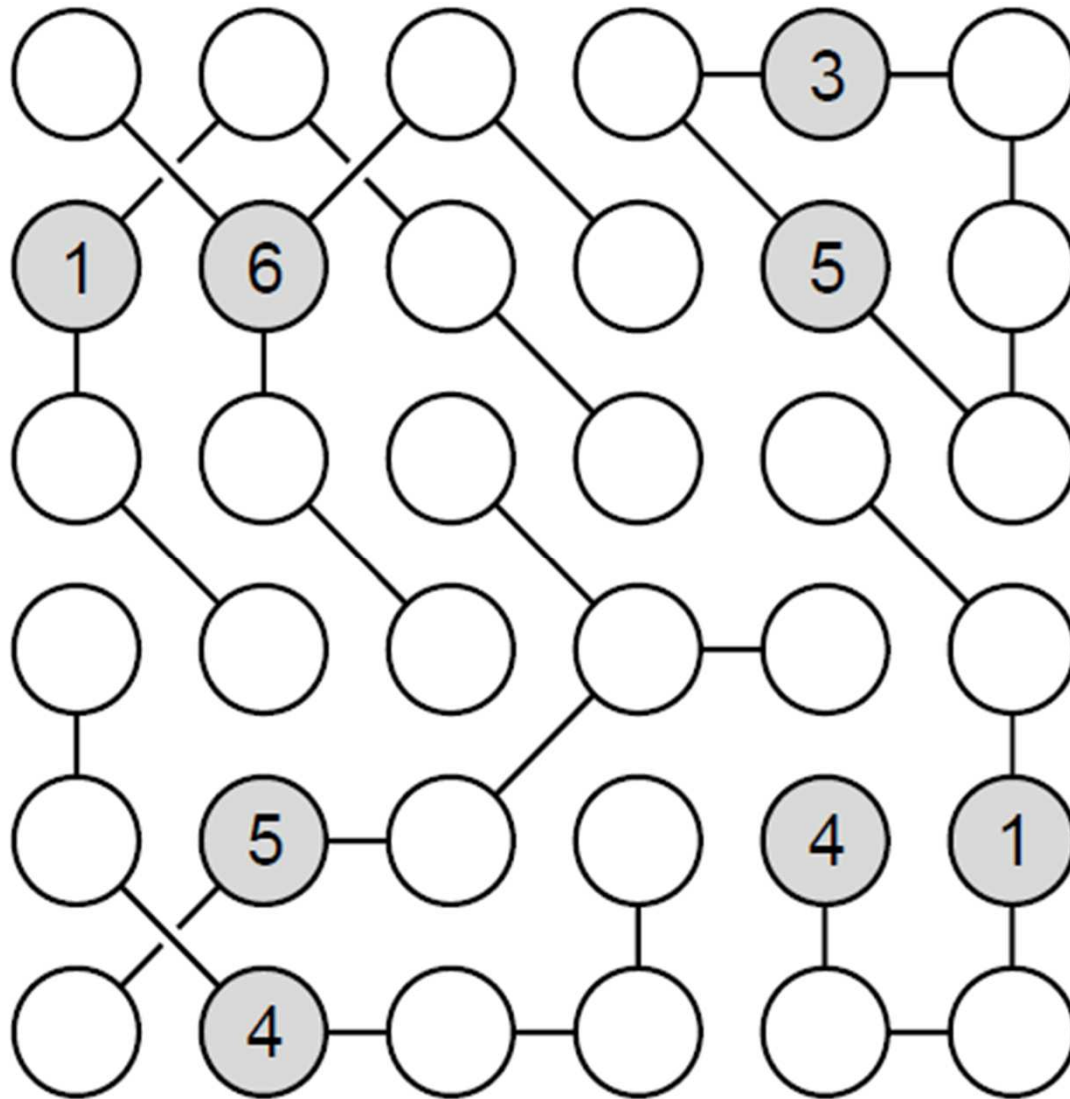
Strimko

A man leaves his camp by traveling due north for 1 mile. He then makes a right turn (90 degrees) and travels due east for 1 mile. He makes another right turn and travels dues south for 1 mile and finds himself precisely at the point he departed from, that is, back at his campsite. Where is the campsite located (or where on earth could such a sequence of events take place)?

Go to Extremes

Manipulate the problem space

Look at extreme limits of the problem space.

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Two flagpoles are standing, each 100 feet tall. A 150-foot rope is strung

from the top of one of the flagpoles to the top of the other and hangs freely

between them. The lowest point of the rope is 25 feet above the ground.

How far apart are the two flagpoles?

Hint: Start by drawing pictures.

What is the length of k?

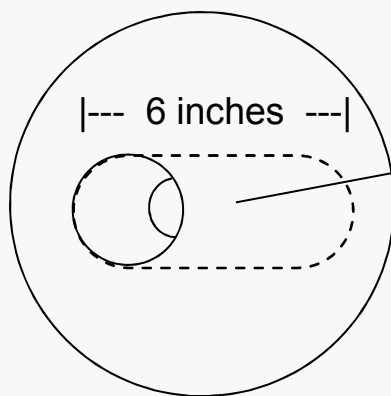Important fact: k remains the same no matter what rectangle is inscribed.
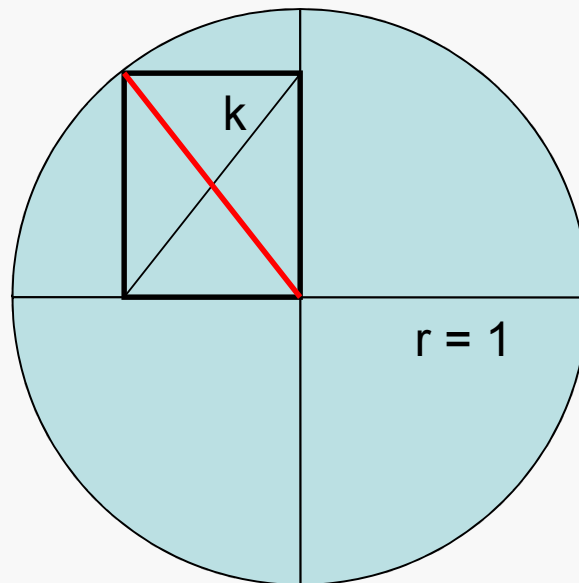
You have a large, solid sphere of gold. A cylinder of space is "bored" through the center of this sphere, producing a ring. The length of that cylindrical line is 6 inches. You want to know how much gold you have left in the ring. Specifically, what is the volume of the ring?

Note: for any sphere,

$$V = \pi D^3/6.$$

|--- 6 inches ---|

Drilling produces an inner cylinder of space 6 inches in length

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Take a number of several digits (say 7 or 8 digits). Reverse it and calculate

the difference. Now if you tell me all but one of the digits in the answer (in

any order), I can tell you the missing digit.

How can you go about figuring out the method?

- You can try some examples and look for a pattern.
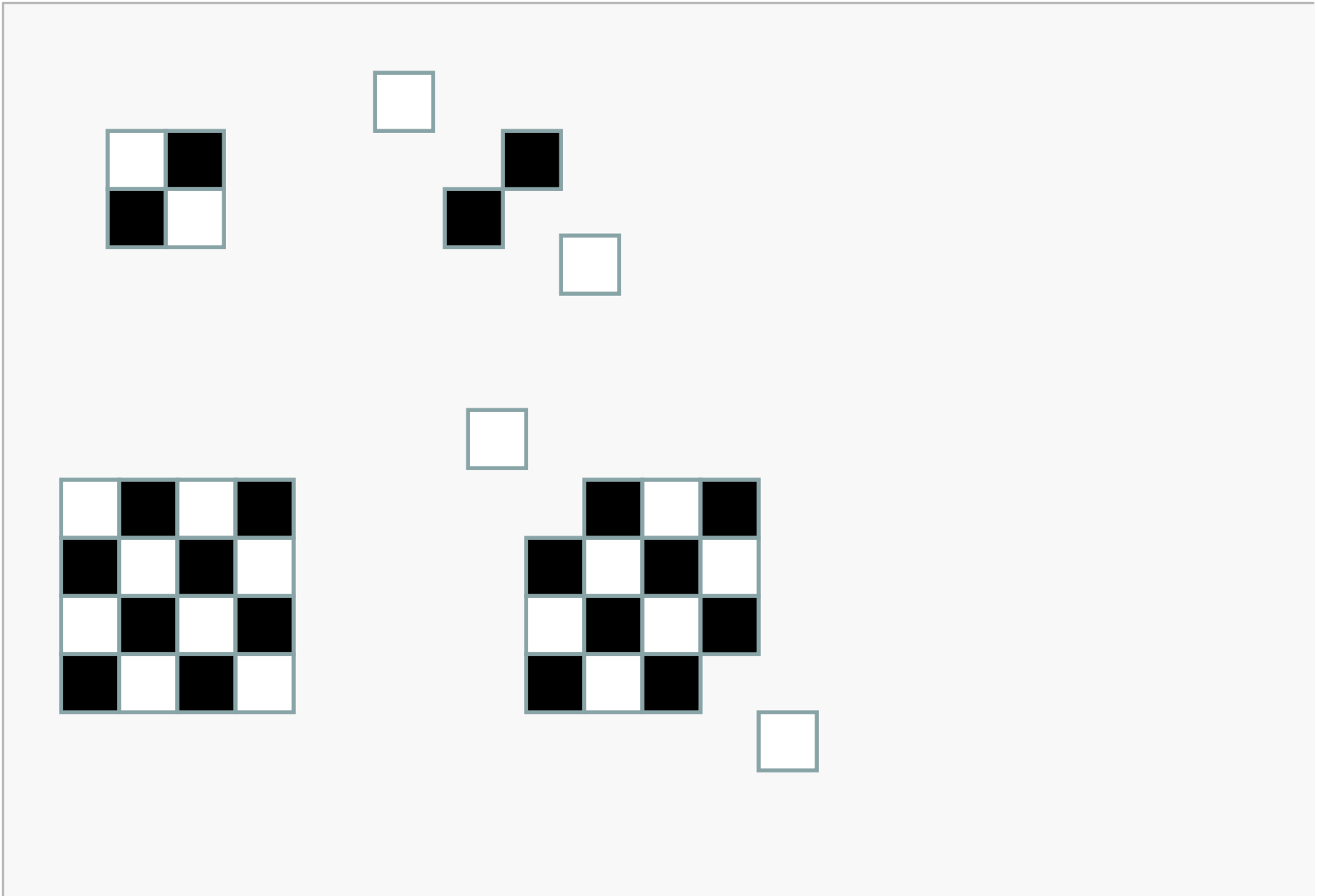- But if you do it on big numbers, it will be hard to figure out.

A domino covers two squares of a chessboard.

1. Can a chessboard be covered by dominos without any dominos sticking out?
2. Now, cut off the upper-left and lower-right corners of the chessboard. Can it still be covered by dominos completely?
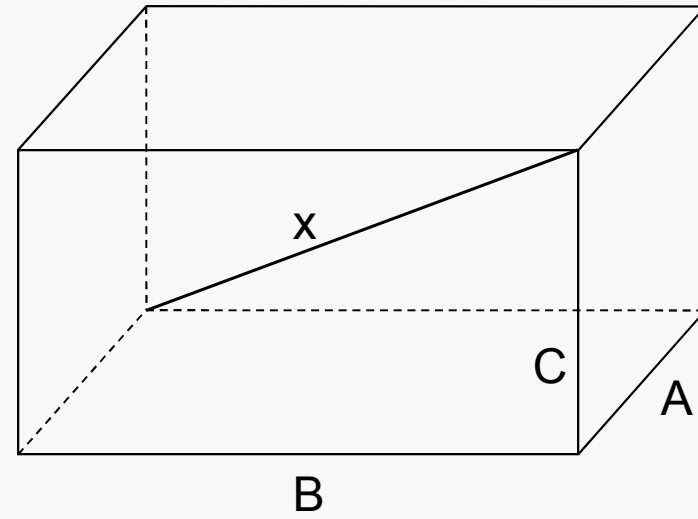
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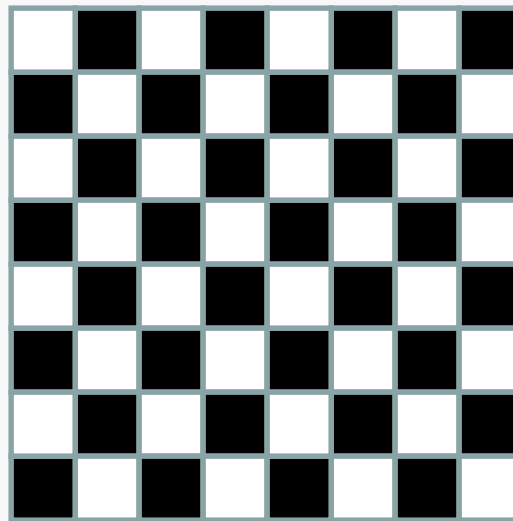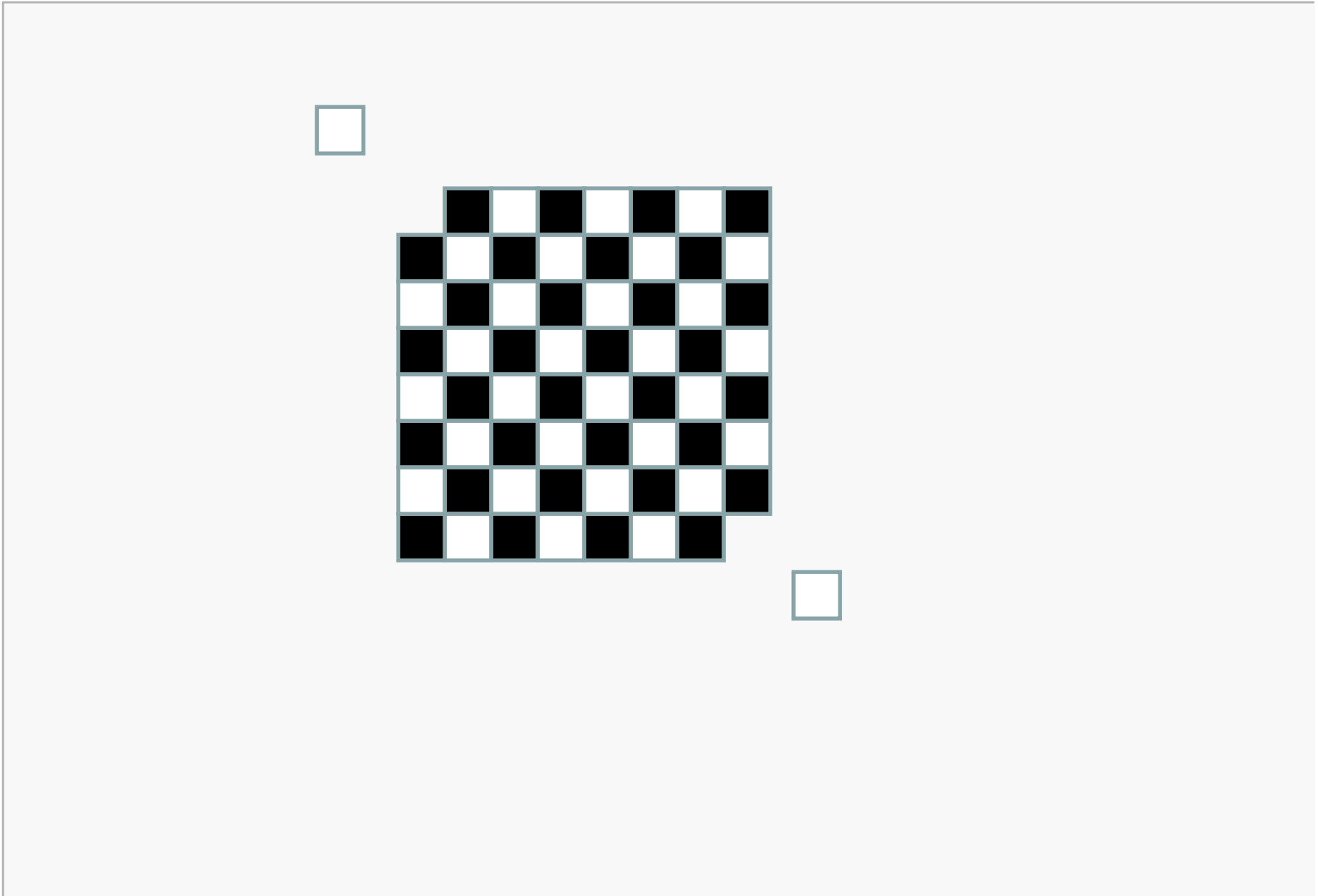
Find the Diagonal

You are given A, B, C. Calculate X.

What is the simpler problem?

How does it relate?

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You have 24 coins that look alike. With the exception of one counterfeit, they are all made of gold and weigh exactly the same. The "bad" coin is either heavier or lighter than the others, you do not know which. You also have available an old-fashioned balance scale.

In the worst case, what is the minimum number of weighings you must make in order to locate the bad coin?

You are given 10 stacks of what should be 10 gold pieces each. Each gold piece weighs two ounces. Unfortunately, one stack contains 10 counterfeits, each coin weighing only one ounce. You have a kitchen-type scale that reads out the weight of what is put on it.

The problem: determine the counterfeit stack with a <u>single</u> weighing.