Algorithms

Algorithms are the threads that tie together most of the subfields of computer science.

Something magically beautiful happens when a sequence of commands and decisions is able to marshal a collection of data into organized patterns or to discover hidden structure.

Donald Knuth

Definition

effective method (or procedure)

a procedure that reduces the solution of some class of problems to a series of rote steps which, if followed to the letter, and as far as may be necessary, is bound to:

- always give some answer rather than ever give no answer;
- always give the right answer and never give a wrong answer;
- always be completed in a finite number of steps, rather than in an infinite number;
- work for all instances of problems of the class.

algorithm

an effective method expressed as a finite list of well-defined instructions for calculating a function

Properties of an Algorithm

An algorithm must possess the following properties:

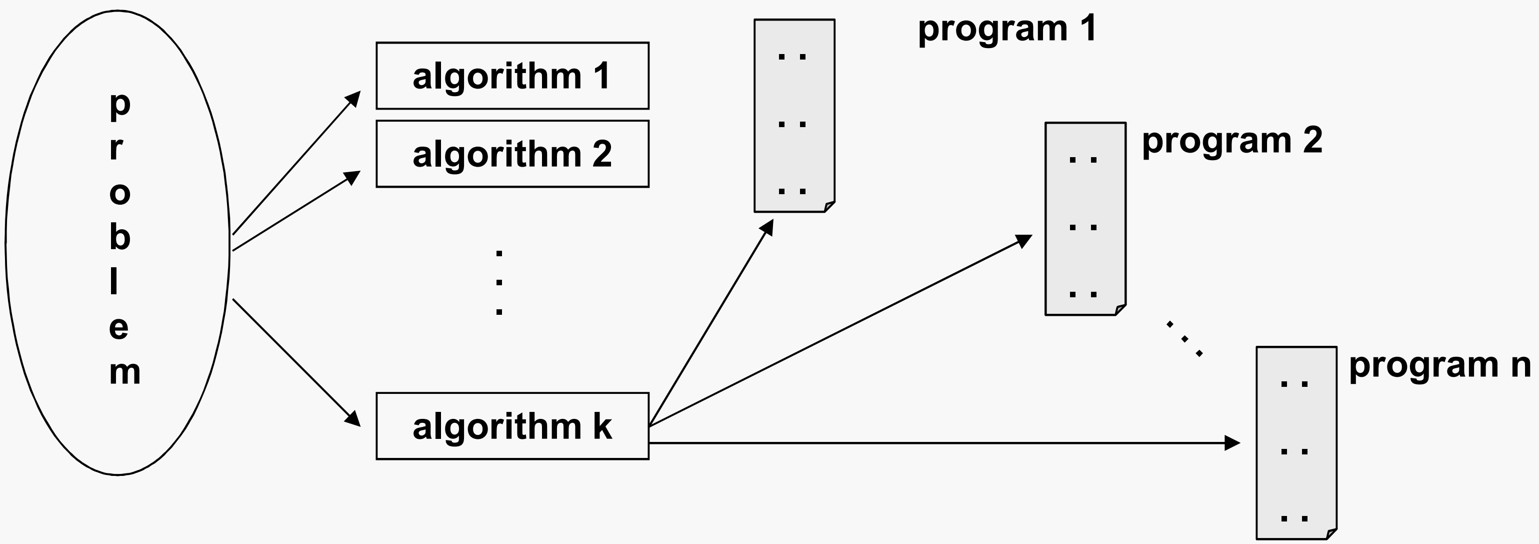
finiteness:	The algorithm must always terminate after a finite number of steps.
definiteness:	Each step must be precisely defined; the actions to be carried out must be rigorously and unambiguously specified for each case.
input:	An algorithm has zero or more inputs, taken from a specified set of objects.
output:	An algorithm has one or more outputs, which have a specified relation to the inputs.
effectiveness:	All operations to be performed must be sufficiently basic that they can be done exactly and in finite length.

Knuth

Problems vs Algorithms vs Programs

For each problem or class of problems, there may be many different algorithms.

For each algorithm, there may be many different implementations (programs).



Expressing Algorithms

An algorithm may be expressed in a number of ways, including:

natural language:	usually verbose and ambiguous
flow charts:	avoid most (if not all) issues of ambiguity; difficult to modify w/o specialized tools; largely standardized
pseudo-code:	also avoids most issues of ambiguity; vaguely resembles common elements of programming languages; no particular agreement on syntax
programming language:	tend to require expressing low-level details that are not necessary for a high-level understanding

Common Elements of Algorithms

acquire data (input)

some means of reading values from an external source; most algorithms require data values to define the specific problem (e.g., coefficients of a polynomial)

computation

some means of performing arithmetic computations, comparisons, testing logical conditions, and so forth...

selection

some means of choosing among two or more possible courses of action, based upon initial data, user input and/or computed results

iteration

some means of repeatedly executing a collection of instructions, for a fixed number of times or until some logical condition holds

report results (output)

some means of reporting computed results to the user, or requesting additional data from the user

pseudo-Language

See the posted notes on pseudo-language notation.

Simple Example: Area of a Trapezoid


```
algorithm AreaOfTrapezoid takes number Height,
                                number lowerBase,
                                number upperBase

# Computes the area of a trapezoid.
# Pre:  input values must be non-negative real numbers.
#

    number averageWidth, areaOfTrapezoid

    averageWidth := ( upperBase + lowerBase ) / 2

    areaOfTrapezoid := averageWidth * Height

    display areaOfTrapezoid
    halt
```

Simple Example: N Factorial

```
algorithm Factorial takes number N

# Computes N! = 1 * 2 * . . . * N-1 * N, for N >= 1
# Pre:  input value must be a non-negative integer.
#
    number nFactorial

    nFactorial := 1

    while N > 1
                nFactorial := nFactorial * N
                N := N - 1
    endwhile

    display nFactorial
    halt
```


Example: Finding Longest Run

```
algorithm LongestRun takes list number List,
                           number Sz

# Given a list of values, finds the length of the longest sequence
# of values that are in strictly increasing order.
# Pre:  input List must contain Sz elements.
#
    number currentPosition   # specifies list element currently
                             #   being examined
    number maxRunLength      # stores length of longest run seen
                             #   so far
    number thisRunLength     # stores length of current run

    if Sz <= 0               # if list is empty, no runs...
       display 0
       halt
    endif

    currentPosition := 1     # start with first element in list
    maxRunLength := 1        # it forms a run of length 1
    thisRunLength := 1

    # continues on next slide...
```


Example: Finding Longest Run

```
# ...continued from previous slide

    while  currentPosition < Sz

        if ( List[currentPosition] < List[currentPosition + 1] )
           thisRunLength := thisRunLength + 1
        else
           if ( thisRunLength > maxRunLength )
              maxRunLength := thisRunLength
           endif
          thisRunLength := 1
        endif

        currentPosition := currentPosition + 1

    endwhile

    display maxRunLength
    halt
```

QTP: is this algorithm correct?

Testing Correctness

How do we know whether an algorithm is actually correct?

First, the logical analysis of the problem we performed in order to design the algorithm should give us confidence that we have identified a valid procedure for finding a solution.

Second, we can test the algorithm by choosing different sets of input values, carrying out the algorithm, and checking to see if the resulting solution does, in fact, work.

BUT… no matter how much testing we do, unless there are only a finite number of possible input values for the algorithm to consider, testing can never prove that the algorithm produces correct results in all cases.

Proving Correctness

We can attempt to construct a formal, mathematical proof that, if the algorithm is given valid input values then the results obtained from the algorithm must be a solution to the problem.

We should expect that such a proof be provided for every algorithm.

In the absence of such a proof, we should view the purported algorithm as nothing more than a heuristic procedure, which may or may not yield the expected results.

Measuring Performance

How can we talk precisely about the "cost" of running an algorithm?

What does "cost" mean? Time? Space? Both? Something else?

And, if we settle on one thing to measure, how do we actually obtain a measurement that makes sense?

This is primarily a topic for a course in algorithms, like CS 3114 or CS 4104.

