This assignment may be done individually. Prepare your answers to the following questions in a plain ASCII text file. Submissions in any other format will not be graded. Submit your file to the Curator system by the posted deadline for this assignment. No late submissions will be accepted.

These questions require you to make clear arguments to specific conclusions, so your score will depend in large part on the completeness and clarity of your answers. Write well.

You will submit your answers to the Curator System (www.cs.vt.edu/curator) under the heading `OOC10`.

1. [20 points] Let O be the origin of the xy-plane. Suppose five more points on the xy-plane, none the same as O, are selected. Prove that there must be two points, call them P and Q, among those five points, such that the angle $\angle POQ$ is acute. Hint: apply the Pigeonhole Principle.

2. [30 points] During a tour of the Wonka Chocolate Company®, the tour group is presented with a Wonka SuperBar®, which is an $m \times n$ rectangular slab composed of equal-sized sections, separated by break lines. It is your job to separate sections of the Wonka SuperBar® and give one of the $m \times n$ sections to each member of the group. You may pick up any piece of the bar that consists of more than one section and separate that piece into two pieces along any vertical or horizontal break line that goes from one side of the piece to the other. Fortunately there are $m \times n$ people in the tour group.

a) What is the smallest number of breaks you can make to supply each member of the group with one section? Why?

b) What is the largest number of breaks you can make to supply each member of the group with one section? Why?

3. [30 points] A group of N people are standing in a large field, in such a way that no pair of people is the same distance apart as any other pair of people. Each of the N people is holding a numbered ball. All of the people are now told to toss the ball they are holding to the person who is closest to them. They all do so.

a) Prove that if N is odd then there is one person to whom no one tosses a ball.

b) Is the same event possible if N is even? Why?

4. [10 points] There are 10,000 different four-digit telephone numbers in the dorms on the VT campus, and more than half of them are located in rooms that are higher than the second story of the building in which they are located. Prove that there must be a telephone number located in a room above the second story in its building that is the sum of two other telephone numbers that are also located in rooms above the second store in their building(s).

5. [10 points] A collection of 27 disks are laid on a table in a rectangular arrangement of 9 rows and three columns. Each disk is either red or blue. Prove that there is a rectangle, with sides running along rows and columns of the arrangement of disks such that the four disks at the corners of the rectangle are all the same color.