Logic is the anatomy of thought.

John Locke

When introduced at the wrong time or place, good logic may be the worst enemy of good teaching.

George Polya

Propositions

A *proposition* is a statement that is true or false:

Di-hydrogen monoxide is liquid at a temperature of 0º Fahrenheit.

The Gregorian calendar uses twelve months.

Every even integer greater than 2 can be expressed as the sum of two prime integers.

Not all declarative statements are propositions:

Fried chicken livers taste wonderful.

This sentence is false.

Propositional logic is concerned with the "algebra" of propositions.

We may form new propositions from old ones by performing certain "algebraic" operations. Consider the following two rather dull propositions:

51 is a prime integer.

The set {3, 5, 7, 11} contains only prime integers.

Then the following are also propositions:

51 is <u>not</u> a prime integer.

51 is a prime integer <u>or</u> the set {3, 5, 7, 11} contains only prime integers.

51 is a prime integer <u>and</u> the set {3, 5, 7, 11} contains only prime integers.

And so are the following:

If 51 is a prime integer, then the set {3, 5, 7, 11} contains only prime integers.

If the set {3, 5, 7, 11} contains only prime integers, then 51 is a prime integer.

If the set {3, 5, 7, 51} contains only prime integers, then 51 is a prime integer.

Logical Connectives

Suppose that P and Q are propositions, then so are the following:

not P	*negation*
P and Q	*conjunction*
P or Q	*disjunction*
if P then Q	*implication*

True or False?

Suppose that P and Q are propositions, then:

not P	is true if and only if P is false
P and Q	is true if and only if P is true and Q is true
P or Q	is true if and only if P is true or Q is true or both are true
if P then Q	is true if and only if P is false or Q is true

Notes:

- "or" is *inclusive*; that is it's true as long as at least one "side" is true
- if… then is false only in the case that the *antecedent* is true and the *consequent* is false
- if… then has absolutely nothing to do with causation

Inference Rules

An *inference rule* is rule that allows us to infer a conclusion from given conditions (the *premise*).

Inference rules take the following general form:

Premise #1.
Premise #2.
. . .
Premise #N.

Conclusion.

For our purposes, it is sufficient to imagine inference rules always have the property that whenever (all of) the premises are true then the conclusion will also be true.

An Inference Rule

Here's one of the most basic inference rules (we assume that P and Q represent specific propositions):

if P then Q.
P.
----------------- *Affirming the antecedent (Modus ponens)*
therefore, Q.

For example, if are given that:

If Cirrus is a cat, then Cirrus is carnivorous.
Cirrus is a cat.

Then we may infer: Cirrus is carnivorous.

Validity and Soundness

An argument is *valid* if the conclusion is true whenever the premises are true.

An argument is *sound* if it is valid and the premises are, in fact, true.

The following argument is valid, but not sound:

> *Premises:*
> If January has 35 days then January contains 5 Mondays.
> January has 35 days.
> *Conclusion:*
> January contains 5 Mondays.

Propositional logic is largely concerned with the issue of how to form valid arguments. The soundness of an argument generally depends upon external (domain-specific) knowledge.

Validity

Note that whether an argument is valid has nothing whatsoever to do with whether its premises are true or its conclusion is true:

Premises:

If Socrates was bipedal, then Socrates was a philosopher.
Socrates was bipedal.

Conclusion (via Affirming the antecedent):

Socrates was a philosopher.

Premises:

If Socrates was from Rome, then Socrates spoke Latin.
Socrates was from Rome.

Conclusion:

Socrates spoke Latin.

A Few More Inference Rules

Again, assume that P and Q represent specific propositions:

if P then Q.
not Q.
--------------------- *Denying the consequent (Modus tollens)*
therefore, not P.

P or Q.
not P.
--------------------- *Disjunctive syllogism (Modus tollendo ponens)*
therefore, Q.

not not P.
--------------------- *Double negation*
therefore, P.

P and Q.
--------------------- *Conjunction elimination*
therefore, P.

Justification

Why is an inference rule accepted?

More specifically, how do we know that an inference rule is valid?

The simple answer is that we may directly argue that if the premises are true then the conclusion must also be true:

P or Q.
not P.
--------------------- *Modus tollendo ponens*
therefore, Q.

Assume the premises are both true. Then since "not P" is true, P must be false. But if "P or Q" is true, at least one of P and Q must be true. Since we know that P must be false, it follows that Q must be true.

Why do we care?

If we want to derive results from a collection of alleged facts (premises) that we possess, we must understand what will guarantee that out results do, in fact, follow from the premises we are given.

Natural languages, especially English, promote using grammatical constructs that lend themselves to obscuring the underling logic, or lack thereof.

And, in any language, there may be implicit assumptions that are required to justify the stated conclusions. If those assumptions are false, then the argument is unsound, even if it is valid (and appears to be sensible when read without a full understanding of the true logic).

Consider the letter-to-the-editor quoted on the following slide…

Why do we care?

This was taken from the Roanoke Times for January 22, 2011:

Lincoln said the war was about the Union

Re: "Slavery was central to Civil War," Jan. 10 commentary:

When I read this essay, it became apparent the author is either a Yankee or someone boasting about his background in history. Maybe both, for him to go out of the way to put down the South as he did.

Just one question: If slavery was so central to causing the war to prevent Southern independence, why did President Lincoln fire Gen. John Fremont for declaring slaves would be set free after the Battle of Wilson's Creek in August 1861 and make the statement: "This war is being fought for a great national idea, the Union, and the general should not have dragged the Negro into it"?

We'll revisit this issue (and example) later in the course. You might consider the merits (or lack thereof) in the author's argument.

Example

This was taken from the *Puzzlers' Paradise* website:

Zookeeper George was in charge of feeding all of the animals in the morning. He had a regular schedule that he followed every day. Can you figure it out from the clues?

P1: Feedings begin at 6:30 am.
P2: A feeding takes 15 minutes.
P3: The last feeding begins no later than 7:30 am.
P4: The giraffes were fed before the zebras but after the monkeys.
P5: The bears were fed 15 minutes after the monkeys.
P6: The lions were fed after the zebras.

A carefully-reasoned analysis will provide a sequence of well-explained inferences, reaching a specific conclusion.

Example

P4: The giraffes were fed before the zebras but after the monkeys.

By Conjunction Elimination, P4 implies:

I1: Monkeys are fed before Giraffes.

I2: Giraffes are fed before Zebras.

P1: Feedings begin at 6:30 am.

P2: A feeding takes 15 minutes.

P1, P2 and I1 imply

I3: Giraffes are fed at least 15 minutes after Monkeys.

P5: The bears were fed 15 minutes after the monkeys.

P1, P2, P5 and I3 imply

I4: Giraffes are fed at least 30 minutes after Monkeys.

I4, P2 and I1 imply

I5: Zebras are fed at least 45 minutes after Monkeys.

Example

P2: A feeding takes 15 minutes.
P6: The lions were fed after the zebras.
I5: Zebras are fed at least 45 minutes after Monkeys.

P2, P6 and I5 imply
I6: Lions are fed at least 60 minutes after Monkeys.

I3 - I6 imply a linear ordering:
I6: Monkeys precede Bears, which precede Giraffes, which precede Zebras, which precede Lions.

P1 - P3 imply
I7: There are five possible feeding times: 6:30, 6:45, 7:00, 7:15 and 7:30.

So I6 and I7 imply that
George feeds Monkeys first at 6:30,
then Bears at 6:45,
then Giraffes at 7:00,
then Zebras at 7:15,
and finally Lions at 7:30.