

Common metaphors for problem solving:

- Moving forward

- Making progress

When you are stuck, how do you move forward?

Hints can help... if you can get one

How do you “give yourself” a hint?

Look for special features in the problem.

$$\begin{array}{r} \\ \\ + \\ \hline \\ \end{array}$$

Standard rules:

- Letters consistently map to numbers

- No leading zero (common use of numbers)

- The numbers must work to add up correctly

What is special here, to get us started?

Another Example

Problems 3

```
      L E T S  
+   W A V E  
-----  
L A T E R
```

Another Crypt-Arithmetic Problem

Problems 4

 D O N A L D
+ G E R A L D

 R O B E R T

```

      xx8xx
xxx | xxxxxxxxx
    xxx
      xxxx
      xxx
        xxxx
        xxxx

```

		1				8	9	
	2	7			9		5	
		4		8	2			
	6		9	2		1	4	
				5				
	9	8		6	1		3	
			2	1		4		
	1		7			3	6	
	7	9				2		

A man leaves his camp by traveling due north for 1 mile. He then makes a right turn (90 degrees) and travels due east for 1 mile. He makes another right turn and travels due south for 1 mile and finds himself precisely at the point he departed from, that is, back at his campsite. Where is the campsite located (or where on earth could such a sequence of events take place)?

Manipulate the problem space

Look at extreme limits of the problem space.

Two flagpoles are standing, each 100 feet tall. A 150-foot rope is strung from the top of one of the flagpoles to the top of the other and hangs freely between them. The lowest point of the rope is 25 feet above the ground. How far apart are the two flagpoles?

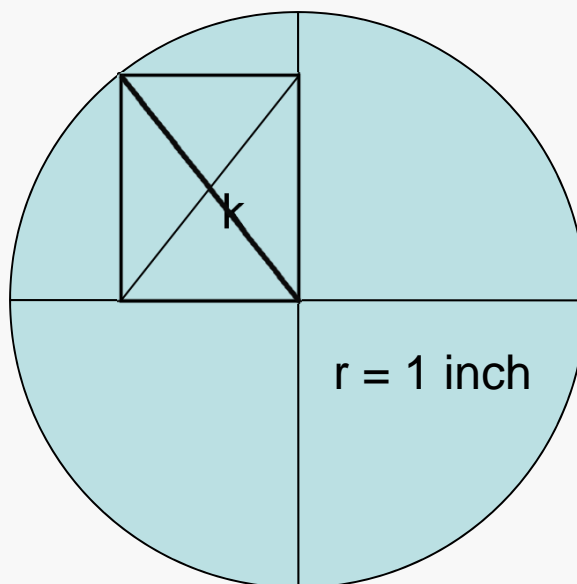
Hint: Start by drawing pictures.

Example

Problems 10

What is the length of k ?

Important fact: k remains the same no matter what rectangle is inscribed.



You have a large, solid sphere of gold. A cylinder of space is “bored” through the center of this sphere, producing a ring. The length of that cylindrical line is 6 inches. You want to know how much gold you have left in the ring. Specifically, what is the volume of the ring?

Note: for any sphere,

$$V = \pi D^3/6.$$

Take a number of several digits (say 7 or 8 digits). Reverse it and calculate the difference. Now if you tell me all but one of the digits in the answer (in any order), I can tell you the missing digit.

How can you go about figuring out the method?

- You can try some examples and look for a pattern.
- But if you do it on big numbers, it will be hard to figure out.

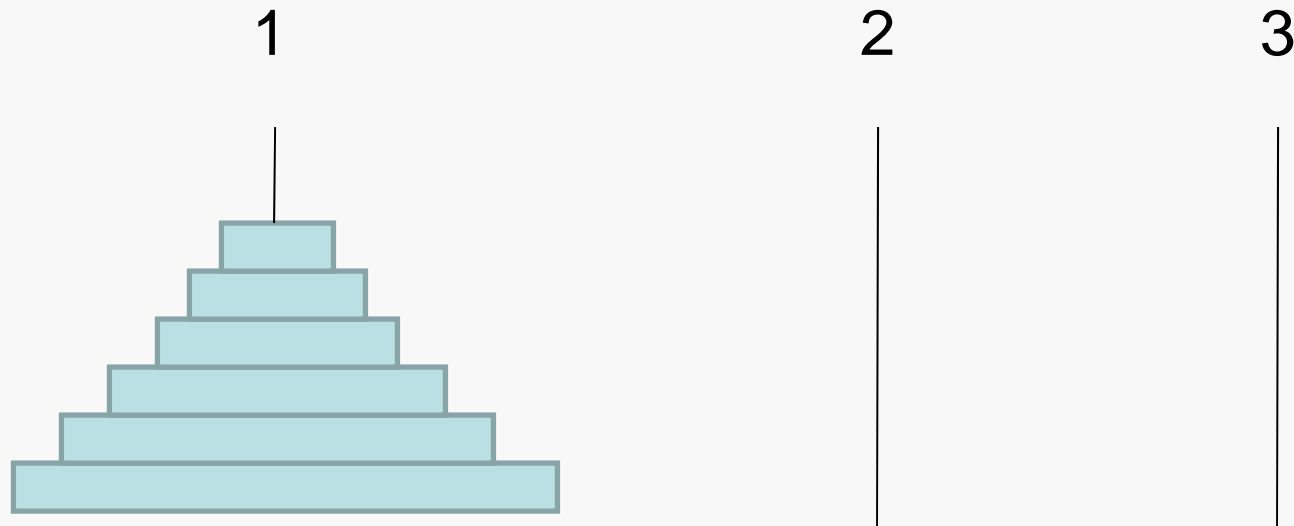
Towers of Hanoi

Problems 13

Move one disk at a time

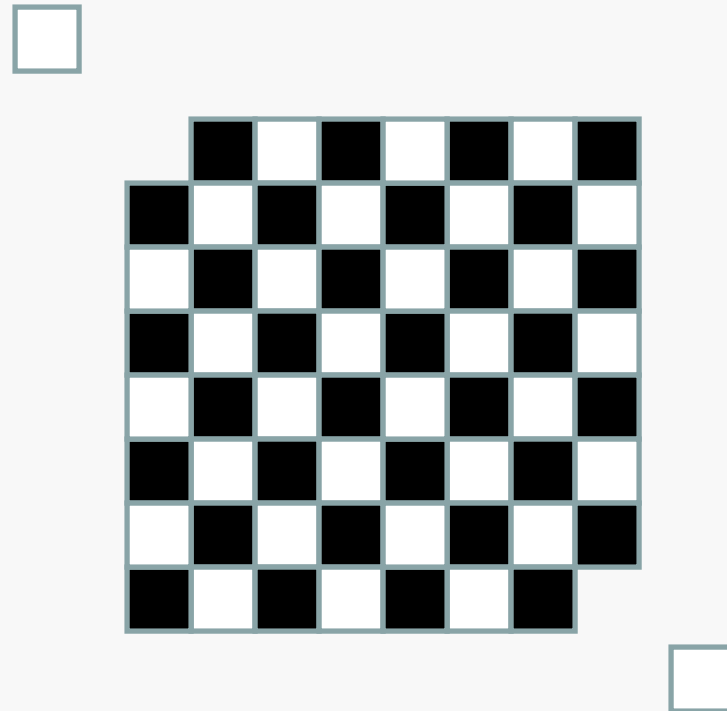
No disk can sit on a smaller disk

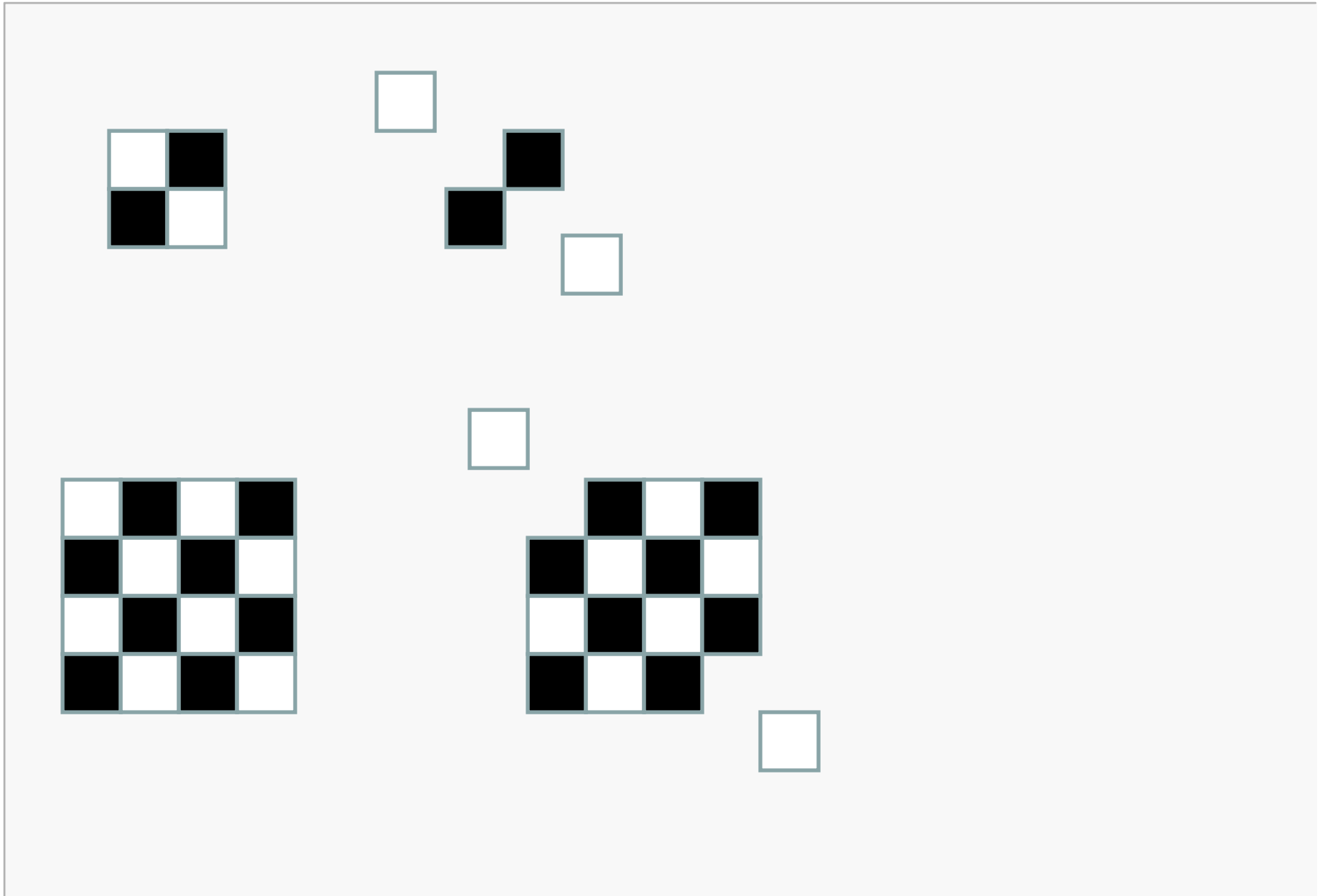
Get all disks from pole 1 to pole 3



A domino covers two squares of a chessboard.

1. Can a chessboard be covered by dominos without any dominos sticking out?
2. Now, cut off the upper-left and lower-right corners of the chessboard. Can it still be covered by dominos completely?





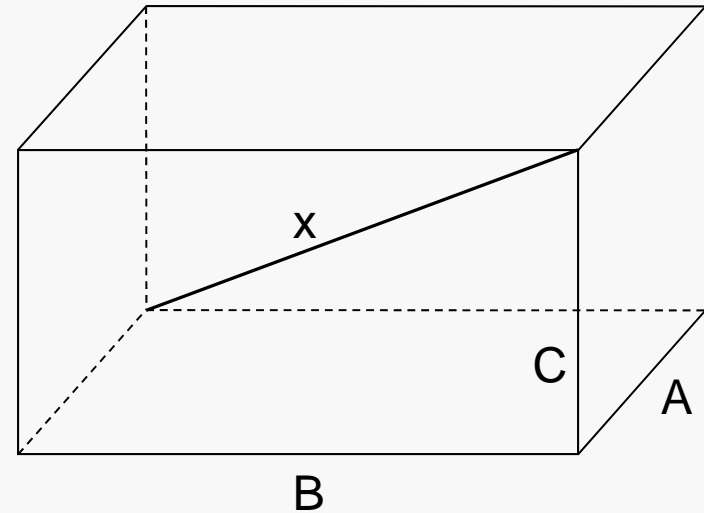
Find the Diagonal

Problems 17

You are given A, B, C. Calculate X.

What is the simpler problem?

How does it relate?



You have 24 coins that look alike. With the exception of one counterfeit, they are all made of gold and weigh exactly the same. The “bad” coin is either heavier or lighter than the others, you do not know which. You also have available an old-fashioned balance scale.

In the worst case, what is the minimum number of weighings you must make in order to locate the bad coin?

You are given 10 stacks of what should be 10 gold pieces each. Each gold piece weighs two ounces. Unfortunately, one stack contains 10 counterfeits, each coin weighing only one ounce. You have a bathroom-type scale that reads out the weight of what is put on it.

The problem: determine the counterfeit stack with a single weighing.