Integer Division

Recall how long division with integers works:

```
          1287     <- Quotient
      +------
  71 | 91434     <- Dividend
Divisor   71000
          -----
          20434
          14200
          -----
           6234
           5680
           ----
            554
            497
            ---     <- Remainder
             57
```

Integer Division

So, the result can be expressed as either of the following forms:

```
91434             57
-----  = 1287 + --
  71              71
```

Dividend Divisor Quotient Remainder

```
91434 = 71 * 1287 + 57
```

The second form corresponds to a most basic result in Number Theory…

Division Algorithm

Given any two integers A and B, where B is not zero, there exist unique integers Q and R such that

$$A = B * Q + R$$

and $0 <= R < B$.

Note: most programming languages have an operation to compute the remainder when integer division is performed (e.g., % in C); most programming languages get this wrong in cases where one of the given integers is negative (by reporting a negative value for the remainder).

Modular Arithmetic

Given any two integers A and B, where B is not zero, then A mod B is defined to be the remainder obtained when A is divided by B.

For example:

17 mod 5 = 2

12 mod 6 = 0

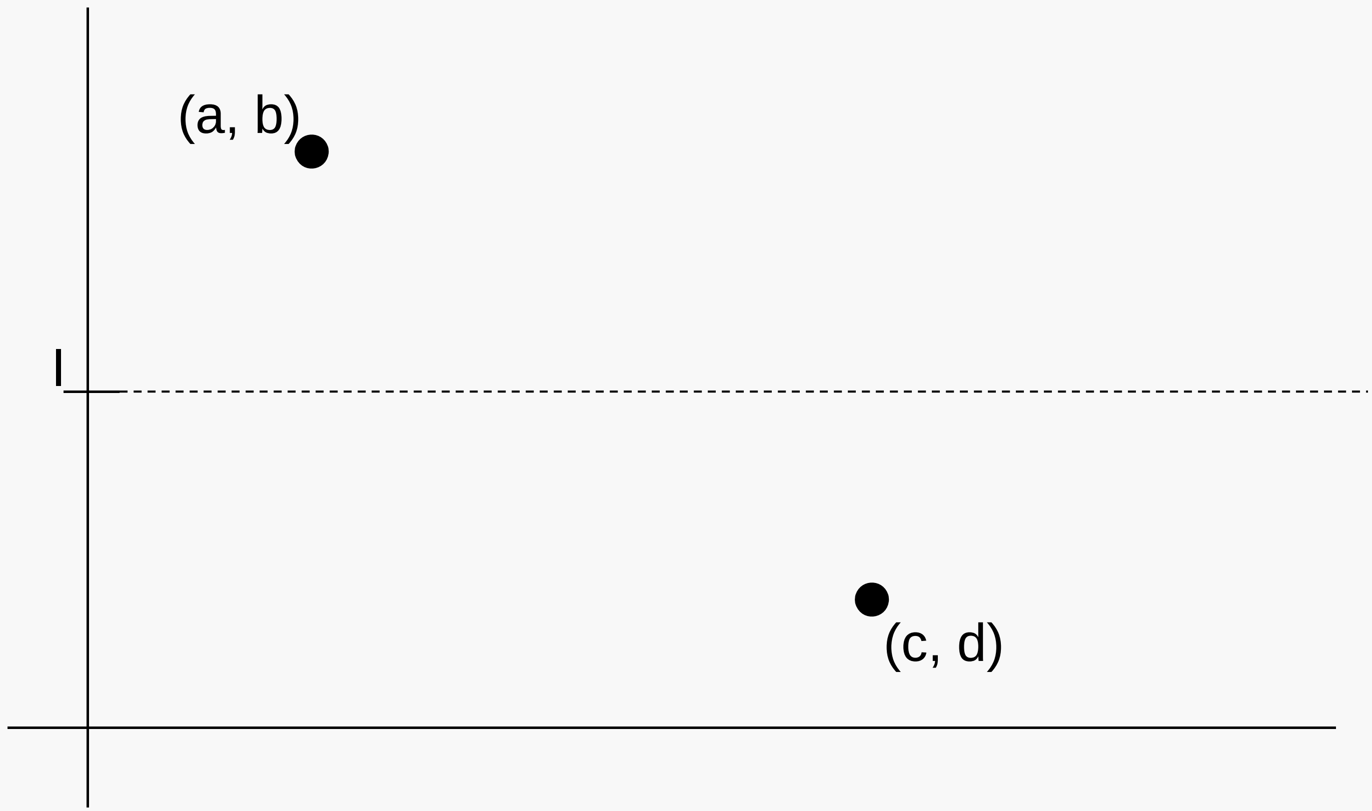
21 mod 50 = 21

Modular arithmetic has many interesting properties and useful applications, and Discrete Mathematics provides a good coverage of the most important aspects.

For our purposes, the basic definitions should be (mostly) sufficient.

Intermediate Value Theorem

Given two points in the XY plane, say (a, b) and (c, d) as shown below, and a value I between b and c, it is impossible to draw a continuous line from the first point to the second point that does not include at least one point with Y-coordinate I:



Counting: the Multiplication Rule

Suppose a sequence of N <u>independent</u> choices must be made, and that the number of different ways to make each choice is C_k.

Then the number of different ways to make the entire sequence of N choices is

$$C_1 * C_2 * \ldots * C_N$$

For example, if email PIDs are composed of only letters and digits, and have a mandatory length of 8 characters, then the number of different PIDs would be

$$36*36*\ldots*36 = 36^8 = 2{,}821{,}109{,}907{,}456$$

And, if the first character is not allowed to be a digit, then the number of different PIDs would be

$$26*36*\ldots*36 = 26*36^7 = 2{,}037{,}468{,}266{,}496$$

Counting: the Addition Rule

Suppose a collection of N <u>independent</u> choices are available, and that the number of different ways to make each choice is C_k.

Then the number of different ways to make a single one of the N choices is

$$C_1+C_2+\ldots+C_N$$

For example, suppose that you are trying to decide whether to order a serving of ice cream or a serving of cake or a serving of pie, and that there are 33 flavors of ice cream, 8 kinds of cake, and 6 kinds of pie.

Then, the total number of ways you can make your choice is

$$33 + 8 + 6 = 47$$

Counting: Permutations

Given a collection of things, an arrangement of those things in a row is called a *permutation*.

Given a collection of N different things, the number of permutations is

$$N! = 1*2*\ldots*N$$

This is really just a special case of the multiplication rule.

Counting: Combinations

Given a collection of N things, an selection of a subset of K those things, in which the order does not matter, is called a *combination of K*.

Given a collection of N different things, the number of combinations of K is

$$\binom{N}{K} = \frac{N!}{K!\,(N-K)!}$$

For example, if you have a poker deck, the number of ways to choose a set of five cards of the same suite (a flush) is

$$\binom{13}{5} = \frac{13!}{5!(8)!} = 1287$$