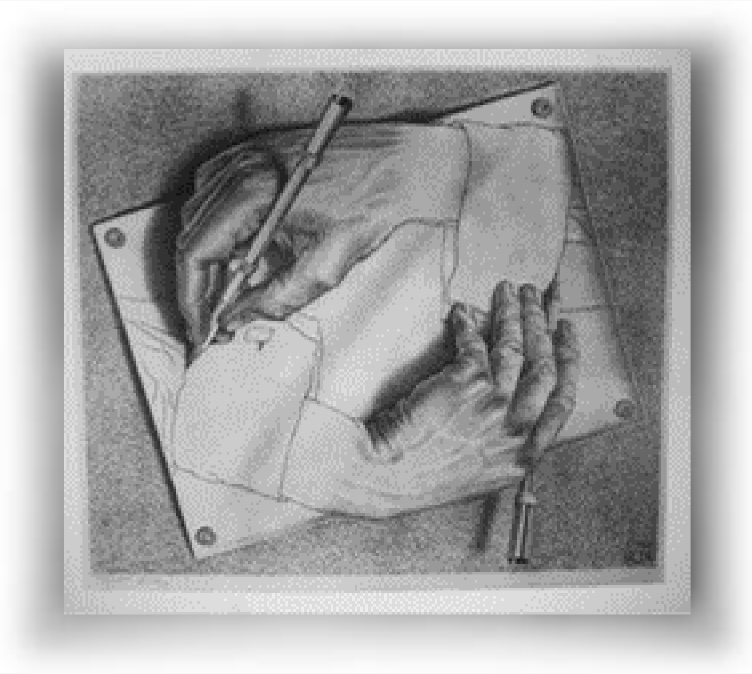
Definitions

Recursion

- see **Recursion**
- a process in which the result of each repetition is dependent upon the result of the next repetition.
- Simplifies program structure at a cost of function calls

Hofstadter's Law

> *"It always takes longer than you expect, even when you take into account Hofstadter's Law."*



Sesquipedalian

> *a person who uses words like sesquipedalian.*

Yogi Berra

> *"Its déjà vu all over again."*

Extended Pseudo-code

To express recursive algorithms, we need to extend the pseudo-code notation to incorporate the notion of an interface to an algorithm:

```
algorithm  <name> takes <list of inputs>
```

We must also be able to express the invocation of an algorithm:

```
<name> ( <list of input values to algorithm> )
```

For some recursive algorithms we need to express algorithm completion communication:

```
return ( <single output value from the algorithm> )
```

Tail Recursion

Tail Recursion: working from the beginning towards the end.

```
# X       list of integers to be summed
# Start   start summing at this index . . .
# Stop    . . . and stop summing at this index
# Pre: X is a list of integers,
       Start & Stop are valid list indexes

algorithm SumArray takes list number X, number Start, number Stop

   if (Start = Stop)                   # base case
      return X[Stop]
   else                                # recursion
      return (X[Start] + SumArray(X, Start + 1, Stop))
   endif
```

Recursive Array Summation Trace

The invocation:

```
List number x
x := [37, 14, 22, 42, 19]
display SumArray( X, 1, 5)
```

would result in the recursive trace:

```
                                         # return values:
SumArray(X, 1, 5)                        #  134

   return(X[1]+SumArray(X,2,5))          #  37 + 97

      return(X[2]+SumArray(X,3,5))       #  14 + 83

         return(X[3]+SumArray(X,4,5) )   #  22 + 61

            return(X[4]+SumArray(X,5,5)) #  42 + 19

               return X[5]               #  19
```

Head Recursion

Head Recursion: working from the end towards the front.

```
# X       list of integers to be summed
# Start   stop summing at this index . . .
# Stop    . . . and start summing at this index
# Pre: X is a list of integers,
       Start & Stop are valid list indexes

algorithm SumArray2 takes list number X, number Start, number Stop

   if (Start = Stop)                   # base case
      return X[Stop]
   else                                # recursion
      return (X[Stop] + SumArray(X, Start, Stop-1))
   endif
```

Recursive Array Summation2 Trace

The invocation:

```
List number x

x := [37, 14, 22, 42, 19]

display SumArray2( X, 1, 5)
```

would result in the recursive trace:

```
                                         # return values:
SumArray2(X, 1, 5)                        #  134

   return(X[5]+SumArray2(X,1,4))          #  19 + 115

      return(X[4]+SumArray2(X,1,3))       #  42 + 73

         return(X[3]+SumArray2(X,1,2) )   #  22 + 51

            return(X[2]+SumArray2(X,1,1)) #  14 + 37

               return X[1]                #  37
```

Middle Decomposition

Middle Recursion: working from middle towards both ends.

```
# X       list of integers to be searched
# Find    integer to be located
# Start   start searching at this index . . .
# Stop    . . . and stop searching at this index
# Pre: X is an ascending ordered list of integers,
#      Find is an integer, Start & Stop are valid list indexes
algorithm BinarySearch takes list number X , number Find,
          number Start, number Stop
   if (Start > Stop)  # base case, value not found
      return -1
   endif

   number mid := trunc( (Start + Stop) / 2 )
   if (Find = list[mid])         # base case
      return mid
   endif
   if (Find < list[mid])         # search lower half
     return BinarySearch(X, Find, Start, mid-1)
   else                          # search upper half
      return BinarySearch(X, Find, mid+1, Stop)
   endif
```

Edges & Center Decomposition

Edges & Center Recursion: working from both ends towards the middle.

Problem:

- sort a subset, (m:n), of an array of integers (ascending order)

Solution:

- Find the smallest and largest values in the subset of the array (m:n) and swap the smallest with the mth element and swap the largest with the nth element, (i.e. order the edges).
- Sort the center of the array (m+1: n-1)

Solution Trace:

	m [0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	n [9]
unsorted list	56	23	66	44	78	99	30	82	17	36

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
after call#1	17	23	66	44	78	36	30	82	56	99

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Variation of the "selection" sort algorithm

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
after call#3	17	23	30	44	56	36	66	78	82	99

Recursive Sorting

```
# ray      list of integers to be sorted
# Start    start sorting at this index . . .
# Stop     . . . and stop sorting at this index
# Pre: ray is a list of integers,
#      Start & Stop are valid list indexes

algorithm DuplexSelection takes list number ray,
          number Start, number Stop

  if (Start < Stop)   #start=stop -> only 1 elem to sort
     number mini := FindMinNumIndex(ray, Start, Stop)
     number maxi := FindMaxNumIndex(ray, Start, Stop)
     SwapEdges( ray, Start, Stop, mini, maxi)
     DuplexSelection( ray, start+1, stop-1 )
  endif
```

Alternatively, the calls to the Find functions can be replaced by a single loop through the list to locate the minimum and maximum indexes.

Recursive Sorting; SwapEdges

```
# ray       list of integers
# Start     left element index
# Stop      right element index
# mini      index for left swapping
# maxi      index for rightswapping
# Pre: ray is a list of integers,
#      Start, Stop mini, maxi are valid list indexes

algorithm SwapEdges takes list number ray,
          number Start, number Stop, number mini, number maxi
  #check for double swap interference
  if ( (mini=Stop) and (maxi=Start) ) #double interference
     Swap( ray, Start, Stop )
  else if (maxi=Start) #low 1/2 interference
         Swap( ray, maxi, Stop )
         Swap( ray, mini, Start )
       else #(mini=Stop) or no interference
          Swap( ray, mini, Start )
          Swap( ray, maxi, Stop )
       endif
  endif
```

Package Shipping Problem

- USPS, FedEx, UPS, DHL, etc.
  - Consider the problem that package shipping companies face:
  - Problem constraints involve:
    - Priority
    - Volume/space : packages/vehicle
    - Weight: packages/vehicle
    - Destination, etc.
  - What if you were hired to write a program to determine which packages should be shipped on a vehicle?
- Simplify
  - When tackling a complex problem, begin by eliminating constraints to focus upon a simpler form of the problem. Then add constraints incrementally to your base solution.
    - For the package problem eliminate all constraints except weight.
    - Do not deal with determining exactly which packages will go on the vehicle.
    - Ignore possible multiple solutions.
    - If an exact solution does not exist, add code for “near” solutions later.
    - The problem simplifies to determining if a subset of a set of values exists that sums to a given total.

Greedy

Greedy Recursion: try every possible solution.

Greedy Algorithms involve **backtracking**. When a possible case has been determined to not be a solution previous work to reach the test for the case will need to be undone.

Knapsack Problem (*very weak form*)

Given an integer total, and an integer list, determine if any collection of list elements within a subset of the list sum up to total.

Algorithm

Check if a collection exists containing the first subset element, (i.e. does collection exist for the remaining elements in the list for the total reduced by the first element)?

If no collection exists containing the first subset element check for a collection for total from subset start + 1 to the end of the subset.

Knapsack Problem

```
# ray       list of integers
# Sum       Subset sum goal
# Start     First subset index
# End       Last subset index
# Pre:      ray is a list of positive integers,
#           Sum is a positive integer
#           Start, End are valid list indexes
algorithm KnapSack takes list number ray,
          number Sum, number Start, number End

  if ( Sum=0 ) #empty collection sums to zero
     return true
  endif
  if ( (Sum<0) or (Start > End) ) #no soln
     return false
  endif
  #check for soln with first element
  if (KnapSack(ray, Sum-ray[Start], Start+1, End) )
     return true
  endif
  #any possible soln cannot contain first element
  return KnapSack(ray, Sum, Start+1, End)
```